University of Nevada, Las Vegas Computer Science 477/677 Spring 2021

Answers to Assignment 1: Due Monday January 26, 2021

1. Problem 0.1 on page 8 of the textbook. In each of the following situations, write O, Ω. Θ in the blank.

   (a) $n - 100 = \Theta(n - 200)$

   (b) $n^{1/2} = O(n^{2/3})$

   (c) $100n + \log n = \Theta(n + \log^2 n)$

   (d) $n \log n = \Theta(10n \log(10n))$
   $n \log n = \Omega(10n + \log(10n))$

   (e) $\log(2n) = \Theta(\log(3n))$

   (f) $10 \log n = \Theta(\log(n^2))$

   (g) $n^{1.01} = \Omega(n \log^2 n)$

   (h) $n^2 / \log n = \Omega(n \log^2 n)$

   (i) $n^{0.1} = \Omega(\log^2 n)$

   (j) $(\log n)^{\log n} = \Omega(n / \log n)$

   (k) $\sqrt{n} = \Omega(\log^3 n)$

   (l) $n^{1/2} = O(5^{log_2 n})$

   (m) $n2^n = O(3^n)$

   (n) $2^n = \Theta(2^{n+1})$

   (o) $n! = \Omega(2^n)$

   (p) $\log n^{\log n} = O(2^{(\log_2 n)^2})$ [hard]

   (q) $\sum_{i=1}^{n} i^k = \Theta(n^{k+1})$

2. Work problem 0.3(c) on page 9 of the textbook.

   $F_n = F_{n-1} + F_{n-2}$ We start by assuming $F_n = 2^{nC}$ for some C. This is false, but it's almost true, that is $\lim_{n\to\infty} \frac{F_n}{2^{nC}} = K = \Theta(1)$ for the correct value of C and some positive number K. Making that assumption:

$$
\begin{aligned}
F_{n+2} &= F_{n+1} + F_n \\
2^{C(n+2)} * K &= 2^{C(n+1)} * K + 2^{Cn} * K \\
\text{Divide both sides by } 2^{Cn} * K : & \\
2^{2C} &= 2^C + 2^0 \\
\text{Substitute } x = 2^C : & \\
x^2 &= x + 1 \\
\text{The quadratic formula gives us two solutions.} & \\
\text{But } x = 2^C \text{ cannot be negative. Thus:} & \\
2^C &= \frac{1+\sqrt{5}}{2} \quad \text{the golden ratio!} \\
C &= \log_2\left(\frac{1+\sqrt{5}}{2}\right)
\end{aligned}
$$

3. Consider the following C++ program.

```
void process(int n)
 {
  cout << n << endl;
  if(n > 1) process(n/2);
  cout << n%2;
 }

int main()
 {
  int n;
  cout << "Enter a positive integer: ";
  cin >> n;
  assert(n > 0);
  process(n);
  cout << endl;
  return 1;
 }
```

The last line of the output of `process(n)` is the binary numeral for n.

4. The recursive algorithm implemented below as a C++ function is used as a subroutine during the calculation of the level payment of an amortized loan. What does it compute?

```
float squre(float x)
 {
  return x*x;
 }

float mystery(float x, int k)
 {
  if (k == 0) return 1.0;
  else if(x == 0.0) return 0.0;
  else if (k < 0) return 1/mystery(x,-k);
  else if (k%2) return x*mystery(x,k-1);
  else return mystery(squre(x),k/2);
 }
```

`mystery(x,k)` returns x^k.