University of Nevada, Las Vegas Computer Science 477/677 Spring 2021

Assignments 4 and 5: Due Tuesday March 9, 2021 11:59 pm

Name:____

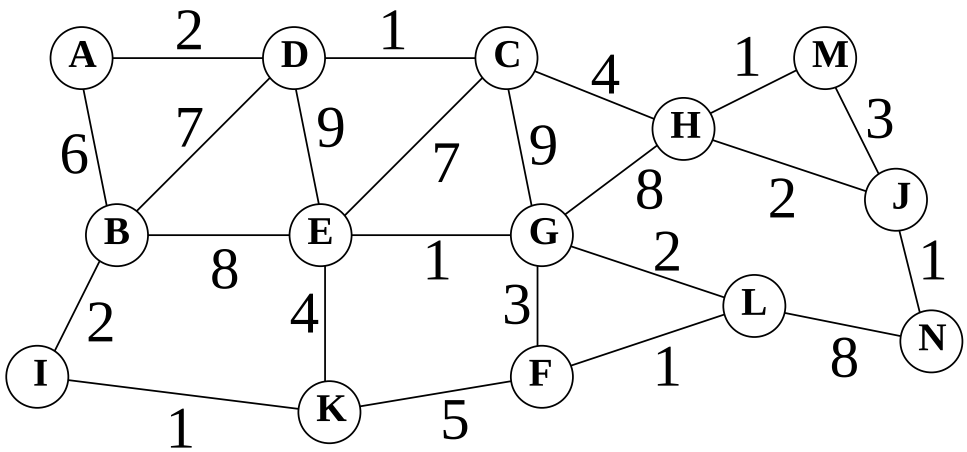
You are permitted to work in groups, get help from others, read books, and use the internet. Turn in the completed assignment on Canvas, using instructions given to you by the grader, Mr. Nicholas Heerdt, by 11:59 PM February 25.

1. Fill in the blanks.
   (a) In ____________________________ hashing, no two data have the same hash value.
   (b) Using ____________________________ coding, the codons for the different symbols of a message may be written consecutively without spaces.

2. Give the asymptotic time complexity in terms of n, using Θ, O, or Ω, whichever is most appropriate.
   (a) $F(n) \geq F(n - \sqrt{n}) + n^2$
   (b) $H(n) < H(n/3) + H(n/4) + 2H(n/5) + n$
   (c) $G(n) = 3(G(2n/3) + G(n/3)) + 5n^2$

3. For each of these recursive subprograms, write a recurrence for the time complexity, then solve that recurrence.
   (a)
   ```
   void george(int n)
    {if(n > 0)
      {for(int i = 0; i < n; i++) cout << "hello" << endl;
       george(n/2); george(n/3); george(n/6);}}
   ```
   (b)
   ```
   void martha(int n)
    {if (n > 1)
     {martha(n-1); martha(n-2):}}
   ```
   Hint: Look at problem 0.3 on page 9 of your textbook.

4. Walk through Kruskal's algorithm to find the minimum spanning tree of the weighted graph shown below. Show the evolution of the union/find structure at several intermediate steps. Whenever there is choice between two edges of equal weight, choose the edge which has the alphabetically largest vertex. Whenever there is a union of two trees of equal weight, choose the alphabetically larger root to be the root of the combined tree. Indicate path compression when it occurs.



5. A hash table with of size $m = 200$ is used to store 400 data items, using a pseudo-random hash function. The average number of items in a cell ("bucket") is clearly $\frac{400}{200} = 2$, but there could be empty cells. What is the expected number of empty cells? Approximate to the nearest integer. Hint: You may need to look in a statistics textbook, or on the internet, to figure this out.

6. Insert the letters B, A, Y, H, P, D into an empty treap, where the "random" keys are given in the following table. Show the treap after each insertion and indicate all rotations.

A	8
B	4
D	10
H	2
P	5
Y	9

7. Find an optimal Huffman code on the alphabet A,B,C,D,E,F where frequencies are given in the following table.

A	8
B	6
C	3
D	11
E	14
F	2

8. A 3-dimensional $9 \times 7 \times 10$ rectangular array A is stored in main memory in row major order, and its base address is 8192. Each item of A takes one word of main memory, that is, one addressed location. Find the address of A[4][5][2].

9. You wish to compute all entries of Pascal's triangle down to the 10th row, namely $C(n,k) = \binom{n}{k}$ using dynamic program and the recurrence $C(n,k) = C(n-1,k-1) + C(n-1,k)$ for $0 < k < n$, and $C(n,0) = C(n,n)$ for all n. To save space, you want to store Pascal's triangle as a triangular array. $C(n,k)$ will be stored as `X[location]`, where `location` is a function of n and k. Here is your code, with one line deleted:

```
int const N = 10;
int X[N*(N+1)/2]; // The size of X is N+1 choose 2, which is 55 if N = 10

 int location(int n,int k)
  {
   assert(0 <= k and k <= n);
   return // FILL IN HERE
  }

  void store(int value, int n, int k)
   {
    X[location(n,k) = Value;
   }

  int fetch(int n, int k)
   {
    return X[location(n,k)];
   }

 int main()
  {
   for(int n = 0; n <= N; n++)
    for(int k = 0; k <= n; k++)
     if(k == 0 or k == n) store(1,n,k);
     else store(fetch(n-1,k-1)+fetch(n-1,k),n,k);
   return 1;
   // Write out the triangle
   for(int n = 0; n <= N; n++)
    {
     for(int k = 0; k <= n; k++)
      cout << " " << fetch(n,k);
     cout << endl;
    }
   return 1;
  }
```

The only missing part is the return value of the function `location`. Fill it in.

10. Consider the following program, where n is a given constant. The purpose of the program is to find the longest increasing subsequence of a sequence.

```
int A[n];
int B[n];
void getA()
 {
  for(int i = 0; i < n; i++) cin >> A[i];
 }
int main()
 {
  getA();
  int s = 0;
  int i = 0;
  while(i < n) // beginning of outer loop
   {
    while(s > 0 and B[s-1] > A[i]) s--; // inner loop
    B[s] = A[i];
    s++;
    i++;
   } // end of outer loop
  for(int j = 0; j < s; j++) cout << B[j];
  cout << endl;
  return 1;
 }
```

(a) If n = 10 and the input stream is 0 6 9 8 1 3 2 5 4 7 what is the output?

(b) The inner and outer loops are both linearly bounded, and thus the time complexity of the code is $O(n^2)$. But, it is not $\Theta(n^2)$. Use amortization to prove that the time complexity is $\Theta(n)$.

I haven't gone over amortization, and besides, my program is incorrect, so this problem will not be graded.

11. You are trying to construct a cuckoo hash table of size 8, where each of the 8 names listed below has the two possible hash values indicated in the array. Using Hall's marriage theorem, prove that you will fail to construct the table.

	h1	h2
Ann	1	0
Bob	4	2
Cal	0	7
Dan	1	6
Eve	1	0
Fay	6	2
Gus	5	3
Hal	0	4

12. Give the asymptotic complexity of $F(n)$ for each of these recurrences, using Θ.

   (a) $F(n) = 2F(n-1) + 1$

   (b) $F(n) = 2F(n-1) + n$

   (c) $F(n) = 2F(n-1) + n^2$

13. Explain how to implement a very sparse one-dimensional array using a search structure.