1. Consider the single source shortest path problem on a weighted digraph $G$. (digraph = directed graph)
   Fill in each blank with one word, or formula. Assume $G$ has $n$ vertices and $m$ arcs (directed edges.)
   Assume that every answer (shortest path from the source to a vertex) has no more than $p$ edges.

   (a) The simple dynamic programming algorithm requires that $G$ be __________. In that case, the
       time complexity of the algorithm is __________.

   (b) The Bellman-Ford algorithm requires that $G$ have no __________ __________. In that case,
       the time complexity of the algorithm is __________.

   (c) Dijkstra’s algorithm requires that $G$ have no __________ __________. In that case, the time
       complexity of the algorithm is __________.

   (d) In order to work the simple dynamic programming algorithm for the single source shortest path
       problem, we must visit the vertices of $G$ in __________ order.

2. int f(int n)
   {
       if(n<7) return 1;
       else return f(n/2)+f(n/2+1)+f(n/2+2)+f(n/2+3);
   }

   The function $f(n)$ can be computed by recursion, as given in the C++ code above. However, we could
   also compute $f(n)$ using dynamic programming, or memoization.

   (a) What is the time complexity of the recursive computation of $f(n)$?

   (b) What is the time complexity of the dynamic programming computation of $f(n)$?

   (c) What is the time complexity of the computation of $f(n)$ using memoization? Hint: Try a large
       value of $n$, such as $n = 1024$. 
3. Let $G$ be a weighted directed graph with $n$ vertices and $m$ arcs (directed edges).

(a) What is the time complexity of the Floyd-Warshall algorithm for the all pairs shortest path problem on $G$?

(b) What is the time complexity of Johnson’s algorithm for the all pairs shortest path problem on $G$?

4. Walk through Dijkstra’s algorithm to solve the single source shortest path problem for the weighted graph shown below, where the source vertex is $S$. Each undirected edge is two directed edges, one in each direction.

Show the set of vertices in the heap, as well as the backpointers, at each step.
5. Write pseudocode for the Floyd-Warshall algorithm on a weighted directed graph $G$. Assume that the vertices of $G$ are the integers from 1 to $n$. We let $W(i, j)$ be the weight of the edge from $i$ to $j$. If there is no such edge, we let $W(i, j) = \infty$ by default. Your output consists of two 2-dimensional arrays. $V(i, j)$ is the least cost of any path from $i$ to $j$, and $\text{back}(i, j)$ is the next-to-the-last vertex on the least cost path from $i$ to $j$.

6. Compute the Levenshtein edit distance between “thourow” and “thorough.” Show the matrix.