University of Nevada, Las Vegas Computer Science 477/677 Spring 2021

Practice for Final Examination: Part II

This portion of the practice final is 240 points.

1. Give the asymptotic time complexity, in terms of n, of each of these code fragments. (10 points each)

   (a)
   ```
   for(int i = 1; i < n; i=2*i)
    for(int j = 1; j < i; j++)
   ```
   (b)
   ```
   for(int i = 1; i < n; i=2*i)
    for(int j = i; j < n; j++)
   ```
   (c)
   ```
   for(int i = 1; i < n; i++)
    for(int j = i; j > 0; j = j/2)
   ```
   (d)
   ```
   for(int i = 1; i < n; i++)
    for(int j = n; j > i; j = j/2)
   ```
   (e)
   ```
   for(int i = 1; i < n*n; i++)
   ```
   (f)
   ```
   for(int i = 1; i*i < n; i++)
   ```
   (g)
   ```
   for(int i = 1; i < n; i++)
    for(int j = 0; j < n; j = j+i)
   ```
   (h) This problem requires two answers. Its time complexity is not Θ of any of the usual functions we deal with. Instead, it's Ω of some function of n and O of some other function of n. Give both.
   ```
   for(int i = 2; i < n; i=i*i)
    for(int j = 1; j < i; j++)
   ```

2. Give asymptotic solutions to the following recurrences.

   (a) $F(n) = F(n/2) + F(n/3) + n$;

   (b) $G(n) = G(n/4) + 2G(n/16) + \sqrt{n}$;

   (c) $H(n) = H(n - \log n) + \log n$

3. [10 points] Draw an acyclic directed graph of 6 vertices and 15 arcs.

4. [10 points] Draw a directed graph with exactly two strong components, each of which has 4 vertices. The graph must have a "source" vertex s from whieh every vertex is reachable.

5. [10 points] Draw a planar graph with 5 vertices and 10 edges.

6. [20 points] Write pseudocode for the Floyd-Warshall algorithm.

Write pseudocode for the Bellman-Ford algorithm. Be sure to encorporate the shortcut.

7. [20 points] If you need to solve the all-pairs problem for a weighted graph with n nodes and m edges, which algorithm would you use?

8. [20 points] Write the Polish and reverse Polish expressions equivalent to $a * (-(b - c) * d)$.

9. [20 points] Prove that there is no comparison-based algorithm for sorting six items that never uses more than nine comparisons.

10. [20 points]

I made a mistake writing this code in Part I of the practice final. Here is the correct version.

```
int product(int a, int b)
 {
  assert(b >= 0);
  int c = a;
  int d = b;
  int total = 0;
  while(d > 0)
   {
    if(d%2) total = total + c;
    c = 2*c;
    d = d/2;
   }
  return total;
 }
```

(a) What does this function do?

(b) What is the loop invariant of the while loop?