Name:________________________________________________________

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided. If you want anything on extra pages to be graded, staple those pages to your test and write, “Please grade this page.”

The entire examination is 240 points.

1. True or False. [5 points each]
   (a) F Open hashing uses open addressing.
   (b) F A good programmer would never use an unordered linked list as a search structure.

2. Fill in the blanks.
   (a) [5 points] Any comparison-based sorting algorithm on a file of size $n$ must execute at least $\log_2 n!$ comparisons in the worst case.
   (b) [5 points] When two data items have the same hash value, that is called a collision.

3. Solve the recurrences. Give asymptotic answers in terms of $n$. [10 points each.]
   (a) $F(n) = 2F(n/3) + n$
      $F(n) = \Theta(n)$
   (b) $F(n) = 4F(n/2) + n$
      $F(n) = \Theta(n^2)$
   (c) $F(n) = F(n - 1) + \log n$
      $F(n) = \Theta(n \log n)$
   (d) $F(n) = 2F(n/4) + \sqrt{n}$
      $F(n) = \Theta(\sqrt{n} \log n)$
   (e) $F(n) = 2F(\sqrt{n}) + \log n$
      $F(n) = \Theta(\log n \log \log n)$
   (f) $F(n) = F(n/2) + F(3n/4) + 3F(n/4) + n^2$
      $F(n) = \Theta(n^2 \log n)$
   (g) $F(n) = 2F(n - 1) + 2^n$
      Hint: let $n = \log_2 m$
      $F(n) = \Theta(n 2^n)$
   (h) $F(n) = F(\log n) + 1$
      $F(n) = \Theta(\log^* n)$
4. [20 points] Explain what a probe sequence is.

In closed hashing, all data are stored in the hash table, no more than one datum per address. Collisions are resolved by open addressing. If an item cannot be stored at the place indicated by its hash value, it searches for an empty location using a probe sequence. A probe sequence is simply a sequence of hash addresses; the item is placed in the first unused address given by that sequence.

5. [30 points] Walk through heapsort, where the input file is as given below.

```
AZGPWFLR
AZGRWFGLP
AZLRWFAGP
AZLRWFAGP
ZALRWFGP
ZWLRAFGP heapification complete
PWLRAGFZ
WPLRAFGZ
WRLPAFGZ
GRLPAFWZ
RGLPAFWZ
RPLGAFWZ
FPLGARWZ
PFLGARWZ
PGLFARWZ
AGLFPRWZ
LGAFPRWZ
FGALPRWZ
GFALPRWZ
AFGLPRWZ
FAGLPRWZ
AFGLPRWZ
```

6. [30 points] You wish to store the entries of a virtual triangular array $A$ in a linear array $B$ in column-major order. $A[i][j]$ exists if and only if $0 \leq i \leq j \leq n$. Find a function $f$ so that $A[i][j]$ is stored at $B[f(i,j)]$ for all $0 \leq i \leq j \leq n$. All indices start at 0, as in C++.

The item $A[i][j]$ is in row $i$ and column $j$, and has two sets of predecessors: $\binom{j}{2}$ in earlier columns and $i$ at earlier positions in column $j$. Thus $f(i,j) = \binom{j}{2} + i = \frac{j(j+1)}{2} + i$

7. [30 points] Use union/find to compute a minimum spanning tree for the graph shown below. Show changes in the union/find forest, including path compression. To make sure everyone who does the problem correctly gets the same answer, use the following tie breaking rule: when you compute the union of two trees of equal size, the larger of the two roots becomes the root of the combined tree.
In the next step, no edge is added to the spanning tree, but \text{find}(A) causes path compression.

8. [30 points] Explain how to use a search structure to implement a sparse virtual array.

Let \( A \) be a virtual array. The search structure \( S \) contains ordered pairs, called \textit{memos}, where the pair \((i, x)\) indicates that \( A[i] = x \). If there is no pair of the form \((i, )\), then \( A[i] = 0 \) (or whatever the default value is.) The \textit{fetch} operator, such as needed in to return the value of \( A[i] \), is implemented by \text{find} in \( S \), which tries to find a memo of the form \((i, x)\) for some \( x \). If such a memo is found, \textit{fetch} returns \( x \), otherwise the default value. \textit{Store} is implemented by inserting a new memo into \( S \), or changing the second term of an existing memo. For example, if \( A[2] = 7 \) is implemented, the memo \((2,7)\) is inserted into \( S \). If there is already a memo \((2, x)\) for some \( x \), that memo is changed to \((2,7)\).