

This is code for an algorithm which finds a maximum length strictly increasing subsequence of a sequence $A[0] \dots A[n-1]$ of integers. There could be more than one such subsequence, but the algorithm chooses just one of them.

In order to explain the algorithm, we define $\text{Length}(i)$ to be the length of the longest strictly monotone subsequence of $A[0] \dots A[i]$. The function Length does not appear in the program, but it is useful for explanation purposes. The arrays `index` and `value` below change during the execution of the program, but after iteration t of the for loop of the function `mainwork()`, `index[ell] = i` and `value[ell] = A[i]` where i is the maximum number no larger than t such that $\text{Length}(i) = \text{ell}$, if such an i exists. If not, `index[ell]` and `value[ell]` are undefined.

I know that's pretty hard to grasp, but it works!

```
// program to find the longest strictly monotone increasing subsequence

int const N = 20;
int n;
int A[N]; // the input sequence
int back[N]; // back[i] = backpointer of longest incs ending at a[i]
int L; // length of longest maximal monotone increasing subsequence so far
int index[N]; // Mystery! (See the definition above.)
int value[N]; // Mystery! (See the definition above.)

void getA()
{
    cin >> n;
    for(int i = 0; i < n; i++) cin >> A[i];
}

void startup()
{
    L = 1;
    index[0] = -1; // this is a fictitious value
    back[0] = -1; // this is a fictitious value
    index[1] = 0;
    value[1] = A[0];
}

void writebackwards(int indx)
{
    if(indx >= 0)
    {
        writebackwards(back[indx]);
        cout << " " << A[indx];
    }
}
```

```

void writeanswer()
{
    writebackwards(index[L]);
    cout << endl;
}

void mainwork()
{
    getA();
    for(int t = 0; t < n; t++)
        cout << " " << A[t]; // print the input sequence
    cout << endl;
    startup();
    for(int t = 1; t < n; t++)
    {
        int s = 1;
        while(s <= L and A[t] > value[s])s++;
        value[s] = A[t];
        index[s] = t;
        back[t] = index[s-1];
        if (s > L) L = s;
    }
    writeanswer(); // print the output sequence
}

int main()
{
    getA();
    mainwork();
    return 1;
}

```

Here are four runs of the program:

```

0 5 9 3 8 7 2 5 9 6 7 3
0 2 5 6 7

```

```

1 1 2 2 3 3
1 2 3

```

```

0 6 9 8 1 3 2 5 4 7
0 1 2 4 7

```

```

1 3 5 7 9 2 4
1 3 5 7 9

```

