

University of Nevada, Las Vegas Computer Science 477/677 Spring 2022

Assignment 2: Due Monday February 7, 2022, Midnight

Name: \_\_\_\_\_

You are permitted to work in groups, get help from others, read books, and use the internet. Please follow Mr. Nicholas Heerd's instructions on how to submit your completed assignment.

Read the handouts recurrence.pdf, linLog.pdf, and bigOThOm.pdf.

1. Solve each recurrence, using  $O$ ,  $\Omega$ , or  $\Theta$ , whichever is appropriate.

(a)  $F(n) = 4F\left(\frac{n}{2}\right) + 5n^2$

(b)  $f(n) = f(n - 1) + n.$

(c)  $f(n) = f\left(\frac{n}{2}\right) + f\left(\frac{n}{3}\right) + n$

(d)  $f(n) = f(\sqrt{n}) + 1.$

(e)  $f(n) = 2f(\sqrt{n}) + \log n$

(f)  $H(n) \leq 2H\left(\frac{n}{2}\right) + n$

(g)  $g(n) = 2g(n - 1) + 1$

(h)  $G(n) \geq G(n - 1) + \lg n$

(i)  $H(n) \leq 2H(\sqrt{n}) + 4.$

(j)  $K(n) = K(n - 2\sqrt{n} + 1) + n.$

(k)  $F(n) \leq F\left(\frac{n}{5}\right) + F\left(\frac{7n}{10}\right) + n$

(l)  $F(n) = 2F\left(\frac{2n}{3}\right) + F\left(\frac{n}{3}\right) + n$

(m)  $f(n) = 1 + f(\log n)$

2. Write the asymptotic time complexity for each code fragment, giving the answer in terms of  $n$ , using  $O$ ,  $\Omega$ , or  $\Theta$ , whichever is appropriate.

(a) 

```
for (int i=1; i < n; i++)
    for (int j=i; j > 0; j--)
        cout << "hello world" << endl;
```

(b) 

```
for (int i=1; i < n; i=2*i)
    for (int j=i; j < n; j++)
        cout << "hello world" << endl;
```

(c) 

```
for (int i=1; i < n; i = 2*i)
    for (int j=1; j < i; j++)
        cout << "hello world" << endl;
```

(d) 

```
for (int i=1; i < n; i++)
    for (int j=1; j < i; j = j*2)
        cout << "hello world" << endl;
```

(e) 

```
for (int i=1; i < n; i++)
    for (int j=i; j < n; j = j*2)
        cout << "hello world" << endl;
```

(f) 

```
for (int i=2; i < n; i = i*i)
    cout << "hello world" << endl;
```

(g) 

```
for (int i=1; i*i < n; i++)
    cout << "hello world" << endl;
```

(h) 

```
for (int i=n; i > 1; i = i/2)
    for (int j=1; j < i; j=2*j)
        cout << "hello world" << endl;
```

(i) This problem differs from the others. There is no  $\Theta$  solution. However, there is an  $O$  solution, and there is an  $\Omega$  solution. Find both.

```
for(int i = 2; i < n*n; i = i*i)
    for(int j = 0; j < i; j++)
        cout << "hello world" << endl;
```

3. Find the asymptotic time complexity, in terms of  $n$ , for each C++ code fragment. Assume  $n \geq 0$ .

```
(a) void f(int i)
    {
        for(int j = 0; j < i; j++)
            cout << "hello world" << endl;
        if(i > 0) f(i/2);
        if(i > 0) f(i/2);
    }
int main()
    {
        f(n);
        return 1;
    }

(b) void f(int i,int j)
    {
        cout << "hello world" << endl;
        if(j > 0)
            f(i,j-1);
        else if(i > 0)
            f(i-1,i);
    }
int main()
    {
        f(n,n);
        return 1;
    }

(c) void f(int i)
    {
        if(i > 0)
        {
            for(int j = 0; j < i*i; j++);
            cout << "hello world" << endl;
            f(2*i/3);
            f(i/3);
            f(2*i/3);
        }
    }
int main()
    {
        f(n);
        return 1;
    }
```