University of Nevada, Las Vegas Computer Science 456/656 Spring 2022

Assignment 4: Due Wednesday March 30 2022

Name:_____

You are permitted to work in groups, get help from others, read books, and use the internet. You will receive a message from our graduate assistant telling you how to turn in the assignment.

Throughout this assignment, you may assume that a language is recursively enumerable if and only if it is accepted by some machine. Recall that "L is recursively enumerable (RE)" means that there is a machine that enumerates L.

1. True/False/Open

   (a) _____ Every subset of a regular langugage is regular.

   (b) _____ If L_1 is $\mathcal{NP}$-complete and L_2 is $\mathcal{NP}$, there is a $\mathcal{P}$-TIME reduction of L_1 to L_2.

   (c) _____ If L_1 is $\mathcal{NP}$-complete and L_2 is $\mathcal{NP}$ and there is a $\mathcal{P}$-TIME reduction of L_1 to L_2, then L_2 is $\mathcal{NP}$-complete.

   (d) _____ If L is $\mathcal{NP}$-complete, there is no polynomial time algorithm which decides L.

   (e) _____ Every $\mathcal{NP}$ language is decidable.

   (f) _____ $\mathcal{NP}$ = co-$\mathcal{NP}$.

   (g) _____ If L_1 is undecidable and there is a recursive reduction of L_1 to l_2, then L_2 is undecidable.

   (h) _____ The CF grammar equivalence problem is recursively enumerable.

   (i) _____ If a language L is decidable, then there must be a machine that enumerates L in canonical order.

   (j) _____ If there is a machine that enumerates a language L, then L must be decidable.

   (k) _____ If there is a machine that accepts a language L, then L must be recursively enumerable (RE).

   (l) _____ If a language L is decidable, there is a machine that enumerates L.

   (m) _____ If there is a machine that enumerates a language L in canonical order, then L must be decidable.

   (n) _____ If $f : \mathcal{N} \to \mathcal{N}$ is a one-to-one and onto function, where $\mathcal{N}$ is the natural numbers (positive integers) we define the *inverse* of f to be a function $g : \mathcal{N} \to \mathcal{N}$ such that $f(g(n)) = n$ and $g(f(n)) = n$ for all $n \in \mathcal{N}$. There exists a one-to-one onto function $f : \mathcal{N} \to \mathcal{N}$ which can be computed in polynomial time whose inverse cannot be computed in polynomial time. (Such a function is called a *one-way* function.)

   (o) _____ There exists a recursive funtion T such that, for any provable statement P, there is a proof of P whose length does not exceed $T(n)$, where n is the length of P.

2. Consider the following CF grammar and LALR parser.

1. $S \to i_2 S_3$
2. $S \to i_2 S_3 e_4 S_5$
3. $S \to w_6 S_7$
4. $S \to a_8$

	ACTION					GOTO
	a	i	e	w	\$	S
0	$s8$	$s2$		$s6$		1
1					halt	
2	$s8$	$s2$		$s6$		3
3			$s4$		$r1$	
4	$s8$	$s2$		$s6$		5
5			$r2$		$r2$	
6	$s8$	$s2$		$s6$		7
7				$r3$	$r3$	
8			$r4$		$r4$	

Walk through the computation of this parser where the input string is *iiwaeia*.

3. Let L be a decidable language. Write a program in pseudo-code that enumerates L in canonical order.

4. Let $L = \{\langle G_1 \rangle \langle G_2 \rangle : G_1, G_2 \text{ are CF grammars that are not equivalent}\}$. Prove that L is recursively enumerable. Assume that the terminal alphabet of both grammars is Σ.

5. Prove that the halting problem is undecidable.

6. Given that 3-SAT is $\mathcal{NP}$-complete, prove, by reduction, that IND, the independent set problem, is also $\mathcal{NP}$-complete.