Assignment 5: Due Wednesday April 6, 2022, Midnight

Name: ____________________________________________

You are permitted to work in groups, get help from others, read books, and use the internet. Please follow Mr. Nicholas Heerdt’s instructions on how to submit your completed assignment.

1. Each answer is either divide and conquer, dynamic programming or greedy.
   (a) Binary search is ____________________
   (b) Huffman’s algorithm is ____________________
   (c) Kruskal’s algorithm is ____________________
   (d) The Bellman-Ford algorithm is ____________________
   (e) Mergesort is ____________________

2. Walk through Kruskal’s algorithm to find the minimum spanning tree of the weighted graph shown below.

![Weighted Graph](image)

3. Find an optimal Huffman code on the alphabet A,B,C,D,E,F where frequencies are given in the following table.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>11</td>
</tr>
<tr>
<td>E</td>
<td>14</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
</tr>
</tbody>
</table>
4. I did almost the same problem as this one on the board, so (almost) all you have to do is remember what I did and write it down.

Consider the function \( f(n) \) computed by the code below.

```c
int f(int n)
{
    // input condition: n >= 0
    if(n<7) return 1;
    else return f(n/2)+f(n/2+1)+f(n/2+2)+f(n/2+3)+n*n;
}
```

The function can be computed by recursion, as given in the C++ code above. However, we could also compute \( f(n) \) using dynamic programming or memoization.

(a) What is the asymptotic value of \( f(n) \)? (The value itself, not the time to compute it.) Write the recurrence.

(b) What is the asymptotic time complexity of the recursive computation of \( f(n) \)? Write the recurrence. (You should be able to solve this problem using one of the theorems we’ve covered, but if you can’t, try programming it for various values of \( n \).)

(c) What is the asymptotic time complexity of the dynamic programming computation of \( f(n) \)? (There is no excuse for not being able to figure this out without writing a program.)

(d) What is the asymptotic time complexity of the computation of \( f(n) \) using memoization? (This is harder than the others. If you write a program and try various values of \( n \), you need a range of large values, like 1024 and up, to get the picture. Remember: memoization uses a sparse array structure.)
5. Consider Johnson’s algorithm on the weighted directed graph $G$ shown below.

The first step is to augment the graph by adding an arc of weight 0 to every vertex from a new source vertex, $s^*$, as shown below.
Using the Bellman-Ford algorithm to compute $h$, label every vertex $v$ with $h(v)$. 
Label every arc of $G$ with the non-negative adjusted weight.

What additional steps would you take to finish Johnson’s algorithm for $G$? (Do not do those steps.)