University of Nevada, Las Vegas Computer Science 477/677 Fall 2021

Answers to Assignment 1: Due Wednesday January 26, 2022

Name:____

You are permitted to work in groups, get help from others, read books, and use the internet. Your answers must be written in a pdf file and uploaded to canvas by midnight January 26th.

1. Problem 0.1 on page 8 of the textbook. Write either O, Ω or Θ in each blank. Do not write O or Ω if Θ is correct.

   (a) $n - 100 = \Theta\left(n - 200\right)$

   (b) $n^{1/2} = O\left(n^{2/3}\right)$

   (c) $100n + \log n = \Theta\left(n + \log^2 n\right)$

   (d) $n \log n = \Omega\left(10n + \log(10n)\right)$

   (e) $\log(2n) = \Theta\left(\log(3n)\right)$

   (f) $10 \log n = \Theta\left(\log(n^2)\right)$

   (g) $n^{1.01} = O\left(n \log^2 n\right)$

   (h) $n^2/\log n = \Omega\left(n \log^2 n\right)$

   (i) $n^{0.1} = \Theta\left(\log^2 n\right)$

   (j) $(\log n)^{\log n} = \Omega\left(n/\log n\right)$

   (k) $\sqrt{n} = \Omega\left(\log^3 n\right)$

   (l) $n^{1/2} = \Omega\left(5^{\log_2 n}\right)$

   (m) $n2^n = O\left(3^n\right)$

   (n) $2^n = \Theta\left(2^{n+1}\right)$

   (o) $n! = \Omega\left(2^n\right)$

   (p) $\log_2 n^{\log_2 n} = O\left(2^{(\log_2 n)^2}\right)$

   (q) $\sum_{i=1}^{n} i^k = \Theta\left(n^{k+1}\right)$

2. Work problem 0.3(c) on page 9 of the textbook.

$F_n = F_{n-1} + F_{n-2}$ We start by assuming $F_n = 2^{nC}$ for some C. This is false, but it's almost true, that is $\lim_{n\to\infty} \frac{F_n}{2^{nC}} = K = \Theta(1)$ for the correct value of C and some positive number K. Making that assumption:

$$\begin{aligned}
F_{n+2} &= F_{n+1} + F_n \\
2^{C(n+2)} * K &= 2^{C(n+1)} * K + 2^{Cn} * K \\
\text{Divide both sides by } 2^{Cn} * K : & \\
2^{2C} &= 2^C + 2^0 \\
\text{Substitute } x = 2^C : & \\
x^2 &= x + 1 \\
\text{The quadratic formula gives us two solutions.} & \\
\text{But } x = 2^C \text{ cannot be negative. Thus:} & \\
2^C &= \frac{1+\sqrt{5}}{2} \quad \text{the golden ratio!} \\
C &= \log_2\left(\frac{1+\sqrt{5}}{2}\right)
\end{aligned}$$

3. Consider the following C++ program.

```
void process(int n)
 {
  cout << n << endl;
  if(n > 1) process(n/2);
  cout << n%2;
 }

int main()
 {
  int n;
  cout << "Enter a positive integer: ";
  cin >> n;
  assert(n > 0);
  process(n);
  cout << endl;
  return 1;
 }
```

The last line of the output of `process(n)` is a string of bits. What does this bitstring represent?

The binary numeral for n.

4. The C++ code below implements a function, "mystery." What does it compute?

```
float squre(float x)
```

```
 {
  return x*x;
 }

float mystery(float x, int k)
 {
  if (k == 0) return 1.0;
  else if(x == 0.0) return 0.0;
  else if (k < 0) return 1/mystery(x,-k);
  else if (k%2) return x*mystery(x,k-1);
  else return mystery(squre(x),k/2);
 }
```

It computes x^k.