

# The Floyd–Warshall Algorithm

UNLV: Analysis of Algorithms

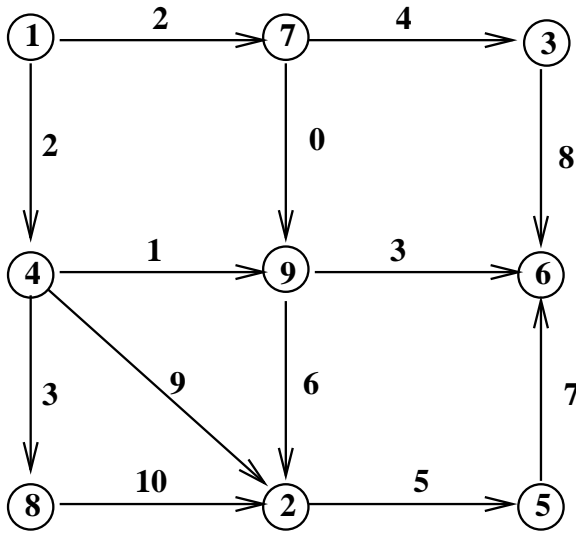
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## The All Pairs Minimum Path Problem

We are given a weighted directed graph  $G = (V, E, W)$ . That is, if  $e = (u, v) \in E$ , then  $W(e) = W(u, v)$  is the *weight* of the edge  $e$ . A solution to the problem consists of arrays  $\{V[u, v]\}_{u, v \in V}$  and  $\{back[u, v]\}_{u, v \in V}$ , such that  $V[u, v]$  is the minimum weight of any path from  $u$  to  $v$ , while  $back[u, v]$  is the next-to-the-last vertex of one of those minimum paths. There is no solution if  $G$  has a negative cycle. For convenience, we assume that the vertices of  $G$  are the integers  $1, \dots, n$ .

```
for all 1 i n
  V[i,i] := 0
  back[i,i] := * \ undefined
endfor
for all 1 <= i <= n
  for all 1 <= j <= n
    if there is an edge from i to j
      back[i,j] := i
      V[i,j] := W(i,j)
    else
      back[i,j] := * \ undefined
      V[i,j] := infinity
    endif
  endfor
endfor
for all 1 <= j <= n
  for all 1 <= i <= n
    for all 1 <= k <= n
      temp := V[i,j] + V[j,k]
      if temp < V[i,k]
        V[i,k] := temp
        back[i,k] := back[j,k]
      endif
    endfor
  endfor
endfor
```

## Output of the Floyd–Warshall Algorithm



We consider the Floyd-Warshall algorithm on the graph illustrated above. The arrays  $V$  and  $back$  are initialized below.

$V$	1	2	3	4	5	6	7	8	9
1	0	$\infty$	$\infty$	2	$\infty$	$\infty$	2	$\infty$	$\infty$
2	$\infty$	0	$\infty$	$\infty$	5	$\infty$	$\infty$	$\infty$	$\infty$
3	$\infty$	$\infty$	0	$\infty$	$\infty$	8	$\infty$	$\infty$	$\infty$
4	$\infty$	9	$\infty$	0	$\infty$	$\infty$	$\infty$	3	1
5	$\infty$	$\infty$	$\infty$	$\infty$	0	7	$\infty$	$\infty$	$\infty$
6	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$	$\infty$	$\infty$
7	$\infty$	$\infty$	4	$\infty$	$\infty$	$\infty$	0	$\infty$	0
8	$\infty$	10	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$
9	$\infty$	6	$\infty$	$\infty$	$\infty$	3	$\infty$	$\infty$	0

$back$	1	2	3	4	5	6	7	8	9
1	$\perp$	$\perp$	$\perp$	1	$\perp$	$\perp$	1	$\perp$	$\perp$
2	$\perp$	$\perp$	$\perp$	$\perp$	2	$\perp$	$\perp$	$\perp$	$\perp$
3	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	3	$\perp$	$\perp$	$\perp$
4	$\perp$	4	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	4	4
5	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	5	$\perp$	$\perp$	$\perp$
6	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
7	$\perp$	$\perp$	7	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	7
8	$\perp$	8	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
9	$\perp$	9	$\perp$	$\perp$	$\perp$	9	$\perp$	$\perp$	$\perp$

What will  $V$  and  $back$  be after one iteration of the outer loop?

We now show  $V$  and  $back$  after two iterations of the outer loop.

$V$	1	2	3	4	5	6	7	8	9
1	0	$\infty$	$\infty$	2	$\infty$	$\infty$	2	$\infty$	$\infty$
2	$\infty$	0	$\infty$	$\infty$	5	$\infty$	$\infty$	$\infty$	$\infty$
3	$\infty$	$\infty$	0	$\infty$	$\infty$	8	$\infty$	$\infty$	$\infty$
4	$\infty$	9	$\infty$	0	14	$\infty$	$\infty$	3	1
5	$\infty$	$\infty$	$\infty$	$\infty$	0	7	$\infty$	$\infty$	$\infty$
6	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	0	$\infty$	$\infty$	$\infty$
7	$\infty$	$\infty$	4	$\infty$	$\infty$	$\infty$	0	$\infty$	0
8	$\infty$	10	$\infty$	$\infty$	15	$\infty$	$\infty$	0	$\infty$
9	$\infty$	6	$\infty$	$\infty$	11	3	$\infty$	$\infty$	0

$back$	1	2	3	4	5	6	7	8	9
1	$\perp$	$\perp$	$\perp$	1	$\perp$	$\perp$	1	$\perp$	$\perp$
2	$\perp$	$\perp$	$\perp$	$\perp$	2	$\perp$	$\perp$	$\perp$	$\perp$
3	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	3	$\perp$	$\perp$	$\perp$
4	$\perp$	4	$\perp$	$\perp$	2	$\perp$	$\perp$	4	4
5	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	5	$\perp$	$\perp$	$\perp$
6	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
7	$\perp$	$\perp$	7	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$	7
8	$\perp$	8	$\perp$	$\perp$	2	$\perp$	$\perp$	$\perp$	$\perp$
9	$\perp$	9	$\perp$	$\perp$	2	9	$\perp$	$\perp$	$\perp$

Fill in the values of the arrays after 6 iterations of the outer loop. (For convenience, do not write  $\infty$  or  $\perp$ ; simply leave the entry blank.)

$V$	1	2	3	4	5	6	7	8	9
1	0								
2		0							
3			0						
4				0					
5					0				
6						0			
7							0		
8								0	
9									0

$back$	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

Fill in the arrays after 8 iterations of the outer loop. (For convenience, do not write  $\infty$  or  $\perp$ ; simply leave the entry blank.)

$V$	1	2	3	4	5	6	7	8	9
1	0								
2		0							
3			0						
4				0					
5					0				
6						0			
7							0		
8								0	
9									0

$back$	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									