

## Strong Components of a Directed Graph

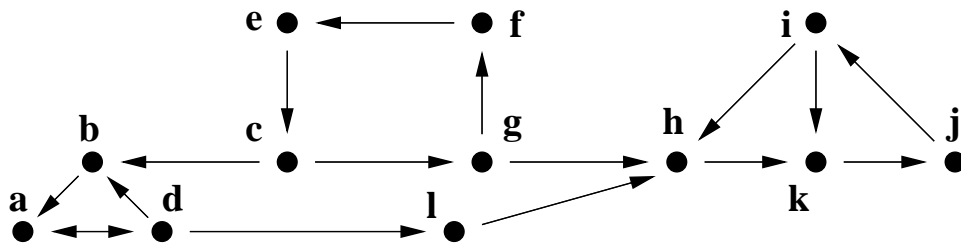
Our textbook, *Algorithms* by Dasgupta, Papadimitriou, and Vazirani, contains what I believe is an important error on page 94, in the description of the algorithm for finding the strong components of a directed graph  $G$ . I believe it should read:

1. Run depth-first search on  $G^R$ , creating a list of the vertices in order of their **post** numbers.
2. Run depth first search on  $G$ , processing the vertices in decreasing order of their **post** numbers from Phase 1.
3. The depth first search in Phase 2 consists of phases. A phase ends when there is no unvisited out-neighbor of the current vertex. The vertices visited during each phase constitute one strong component.

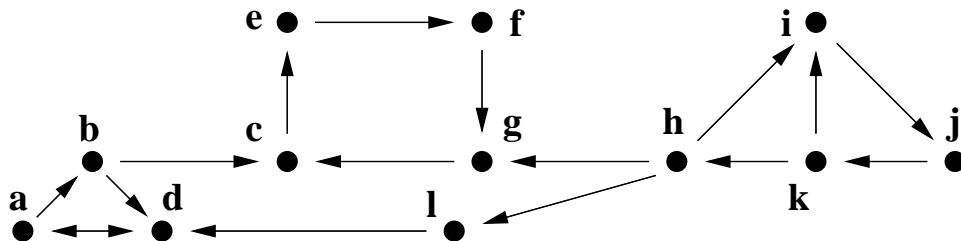
You can reverse these; use  $G$  in Phase 1 and  $G^R$  in step 2. The strong components are exactly the same, but created in a different order.

### An Example

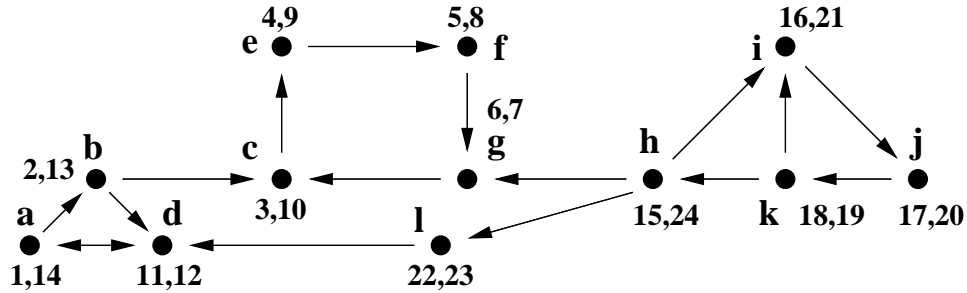
We will step through the algorithm for a directed graph  $G$  of twelve vertices shown below. We use lower case letters a ... l for the names of the vertices.



The reverse graph  $G^R$ :

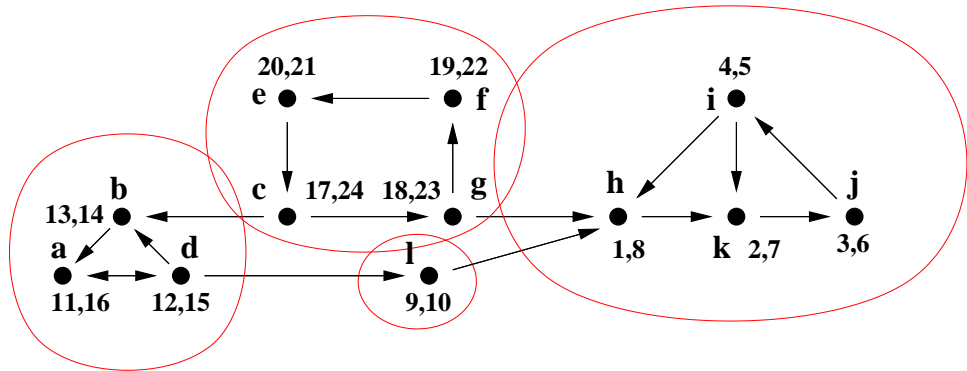


We now execute Phase 1 of the algorithm. Each vertex is labeled with its **pre** and **post** numbers.



At each postvisit, we append the name of the vertex to a list. The list of vertices in order of their Phase 1 post number is  $g, f, e, c, d, b, a, k, j, i, l, h$ .

We now execute Phase 2, processing vertices in the reverse order of our list. (We do not actually use the Phase 2 **pre** and **post** numbers, shown just to aid comprehension.)



We show the stack at each step of Phase 2, where  $\$$  indicates the bottom of the stack. A component is defined whenever the stack becomes empty, after which **explore** begins at the unvisited vertex with the largest Phase 1 **post** number. Strong components are indicated in the figure.

\$	\$ad	
\$h	\$adb	
\$hk	\$ad	
\$hk	\$a	
\$hkj	\$	{a, d, b} is a strong component
\$hkji	\$c	
\$hkj	\$cg	
\$hk	\$cgf	
\$h	\$cgfe	
\$	\$cgf	{h, k, j, i} is a strong component
\$l	\$cg	
\$	\$c	{l} is a strong component
\$a	\$	{c, g, f, e} is a strong component