The entire examination is 250 points.

1. True or False. [5 points each]
   
   (a) F Computers are so fast today that complexity theory is only of theoretical, but not practical, interest.
   
   (b) T \( \log^*(2^n) = \Theta(\log^* n) \)

2. Fill in the blanks.

   (a) [10 points] Any comparison-based sorting algorithm on a file of size \( n \) must execute at least \( \Omega(n \log n) \) comparisons in the worst case. Alternative answer: \( \log_2(n!) \).

   (b) [10 points] Name two well-known divide-and-conquer sorting algorithms.

   quicksort

   mergesort

3. The answer to each of the following two questions is bubblesort, selection sort, or insertion sort.

   (a) [10 points] Treesort is a fast form of insertion sort.

   (b) [10 points] The following C++ code fragment implements selection sort. Assume that A is an array of size n, and that \( \text{swap}(\text{int} & a, \text{int} & b) \) transposes a and b.

   ```
   for(int i = 0; i < n; i++)
   for(int j = i+1; j < n; j++)
   ```

4. Find the time complexity of each of these code fragments in terms of \( n \), using \( \Theta \) notation.

   (a) for(int i = 0; i*i < n; i++)
       \( \Theta(\sqrt{n}) \)

   (b) for(int i = 0; i < n; i++)
       for(int j = 1; j < i; j = 2*j);
       \( \Theta(n \log n) \)

   (c) for(int i = 1; i < n; i++)
       for(int j = i; j < n; j = 2*j);
       \( \Theta(n) \)

   (d) for(float x = n; x > 2.0; x = sqrt(x)) (sqrt(x) returns the square root of x.)
       \( \Theta(\log \log n) \)
(e) for(int i = 1; i < n; i = 2*i) 
    for(int j = 2; j < i; j = j*j);

(Hint: use substitution)
Θ(log n log log n)

5. Solve the recurrences. Give asymptotic answers in terms of n. [10 points each.]

(a) F(n) = F(n/2) + 1
   Θ(log n)
(b) F(n) = 2F(n/2) + 1
   Θ(n)
(c) F(n) = 2F(n/2) + n
   Θ(n log n)
(d) F(n) = n + F(n/5) + F(7n/10)
   Θ(n)
(e) F(n) = F(3n/4) + F(n/2) + 3F(n/4) + n
   Θ(n²)
(f) F(n) = F(n - 1) + √n
   Θ(n³)
(g) F(n) = F(n - log n) + log n
   Θ(n)
(h) F(n) = F(log n) + 1
   Θ(log* n)

6. [20 points] A minheap is implemented as a binary tree, which is implemented as an array, as we saw in class. Suppose the array has 7 items as shown in the first row of the matrix below. Work out the steps of the minheap when the letter B is inserted. (Not all rows are needed.)

```
A C J D F R P
A C J D F R P B
A C J B F R P D
A B J C F R P D
```

7. [20 points] Consider the following procedure:

```cpp
void hold(int n)
{
  if(n >= 1)
  {
    for (int i = 1; i < n; i++)
      cout << "Thank you for holding. Someone will be with you shortly." << endl;
  
```
Consider the question of how many lines of output the execution of `hold(n)` would produce. Write down an appropriate recurrence for this question, and give an asymptotic solution in terms of $n$, using either $O$, $\Omega$, or $\Theta$, whichever is most appropriate.

$$T(n) = 2T(n-1) + n$$

$$T(n) = \Theta(2^n)$$

8. [30 points]

(a) Illustrate the circular linked list implementation of `queue`. Assume that the contents of the queue (from front to rear) is the list Ann, Ted, Sue, Bob.

(b) Illustrate the steps for inserting the item Fay.

(c) Illustrate the steps for deletion of an item, starting from the queue you illustrated for (8a).