University of Nevada, Las Vegas Computer Science 477/677 Spring 2022

Answers to Examination February 14, 2022

The entire examination is 250 points.

1. True or False. [5 points each]

   (a) **F** Computers are so fast today that complexity theory is only of theoretical, but not practical, interest.

   (b) **T** $\log^*\left(2^{2^n}\right) = \Theta(\log^* n)$

2. Fill in the blanks.

   (a) [10 points] Any comparison-based sorting algorithm on a file of size n must execute at least $\Omega(n \log n)$ comparisons in the worst case. Alternative answer: $\log_2(n!)$.

   (b) [10 points] Name two well-known divide-and-conquer sorting algorithms.

   quicksort

   mergesort

3. The answer to each of the following two questions is bubblesort, selection sort, or insertion sort.

   (a) [10 points] Treesort is a fast form of **insertion sort**.

   (b) [10 points] The following C++ code fragment implements **selection sort**. Assume that A is an array of size n, and that `swap(int & a, int & b)` transposes a and b.

```
for(int i = 0; i < n; i++)
  for(int j = i+1; j < n; j++)
    if(A[j] < A[i]) swap(A[i],A[j]);
```

4. Find the time complexity of each of these code fragments in terms of n, using Θ notation.

   (a)
```
for(int i = 0; i*i < n; i++)
```
   $\Theta(\sqrt{n})$

   (b)
```
for(int i = 0; i < n; i++)
 for(int j = 1; j < i; j = 2*j);
```
   $\Theta(n \log n)$

   (c)
```
for(int i = 1; i < n; i++)
 for(int j = i; j < n; j = 2*j);
```
   $\Theta(n)$

   (d)
```
for(float x = n; x > 2.0; x = sqrt(x))
```
   (`sqrt(x)` returns the square root of `x`.)

   $\Theta(\log \log n)$

(e)
```
for(int i = 1; i < n; i = 2*i)
 for(int j = 2; j < i; j = j*j);
```
(Hint: use substitution)
$\Theta(\log n \log\log n)$

5. Solve the recurrences. Give asymptotic answers in terms of n. [10 points each.]

(a) $F(n) = F(n/2) + 1$
$\Theta(\log n)$

(b) $F(n) = 2F(n/2) + 1$
$\Theta(n)$

(c) $F(n) = 2F(n/2) + n$
$\Theta(n \log n)$

(d) $F(n) = n + F(n/5) + F(7n/10)$
$\Theta(n)$

(e) $F(n) = F(3n/4) + F(n/2) + 3F(n/4) + n$
$\Theta(n^2)$

(f) $F(n) = F(n-1) + \sqrt{n}$
$\Theta(n^{\frac{3}{2}})$

(g) $F(n) = F(n - \log n) + \log n$
$\Theta(n)$

(h) $F(n) = F(\log n) + 1$
$\Theta(\log^* n)$

6. [20 points] A minheap is implemented as a binary tree, which is implemented as an array, as we saw in class. Suppose the array has 7 items as shown in the first row of the matrix below. Work out the steps of the minheap when the letter B is inserted. (Not all rows are needed.)

A	C	J	D	F	R	P	
A	C	J	D	F	R	P	B
A	C	J	B	F	R	P	D
A	B	J	C	F	R	P	D

7. [20 points] Consider the following procedure:

```
void hold(int n)
 {
  if(n >= 1)
   {
    for (int i = 1; i < n; i++)
     cout << "Thank you for holding. Someone will be with you shortly." << endl;
```

```
    hold(n-1);
    hold(n-1);
   }
 }
```

Consider the question of how many lines of output the execution of `hold(n)` would produce. Write down an appropriate recurrence for this question, and give an asymptotic solution in terms of `n`, using either O, Ω, or Θ, whichever is most appropriate.

$T(n) = 2T(n-1) + n$

$T(n) = \Theta(2^n)$

8. [30 points]

   (a) Illustrate the circular linked list implementation of **queue**. Assume that the contents of the queue (from front to rear) is the list **Ann, Ted, Sue, Bob**.

   (b) Illustrate the steps for inserting the item **Fay**.

   (c) Illustrate the steps for deletion of an item, starting from the queue you illustrated for (8a).