University of Nevada, Las Vegas Computer Science 477/677 Spring 2023

Answers to Assignment 1: Due Wednesday January 25, 2023

1. Problem 0.1 on page 8 of the textbook. Write either O, Ω or Θ in each blank. Do not write O or Ω if Θ is correct.

   (a) $n - 100 = \Theta\left(n - 200\right)$

   (b) $n^{1/2} = O\left(n^{2/3}\right)$

   (c) $100n + \log n = \Theta\left(n + \log^2 n\right)$

   (d) $n \log n = \Omega\left(10n + \log(10n)\right)$

   (e) $\log(2n) = \Theta\left(\log(3n)\right)$

   (f) $10 \log n = \Theta\left(\log(n^2)\right)$

   (g) $n^{1.01} = \Omega\left(n \log^2 n\right)$

   (h) $n^2/\log n = \Omega\left(n \log^2 n\right)$

   (i) $n^{0.1} = \Omega\left(\log^2 n\right)$

   (j) $(\log n)^{\log n} = \Omega\left(n/\log n\right)$

   (k) $\sqrt{n} = \Omega\left(\log^3 n\right)$

   (l) $n^{1/2} = O\left(5^{\log_2 n}\right)$

   (m) $n2^n = O\left(3^n\right)$

   (n) $2^n = \Theta\left(2^{n+1}\right)$

   (o) $n! = \Omega\left(2^n\right)$

   (p) $\log_2 n^{\log_2 n} = O\left(2^{(\log_2 n)^2}\right)$

   (q) $\sum_{i=1}^{n} i^k = \Theta\left(n^{k+1}\right)$

2. The following problem is a modified version of problem 0.3(c) on page 9 of the textbook. The answer is not exactly the same as the answer to the textbook version, but the techniques are similar.

   $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. We start by assuming $F_n = n^C$ for some constant C. This is false, but it's true in the limit; that is, for larger and larger values of n, the ratio of F_n to

C^n converges to 1. That is, there is a constant C such that $\lim_{n\to\infty} \frac{F_n}{n^C} = 1$. for the correct value of C. Making the assumption that they are actually equal:

$$\begin{aligned} F_{n+2} &= F_{n+1} + F_n \\ C^{n+2} &= C^{n+1} + C^n \\ \text{Divide both sides by } C^n: & \\ C^2 &= C^1 + C^0 \end{aligned}$$

Which is a quadratic equation over C. The quadratic formula gives us two solutions: $C = \frac{1\pm\sqrt{5}}{2}$ But C is not the negative solution, since $\{F_n\}$ would converge to zero. thus $C = \frac{1+\sqrt{5}}{2}$ which is the so-called *golden ratio.* Since we did make an incorrect assumption, we might worry about this answer. But it's correct.

3. Consider the following C++ program.

```
void process(int n)
 {
  cout << n << endl;
  if(n > 1) process(n/2);
  cout << n%2;
 }

int main()
 {
  int n;
  cout << "Enter a positive integer: ";
  cin >> n;
  assert(n > 0);
  process(n);
  cout << endl;
  return 1;
 }
```

The last line of the output of `process(n)` is a string of bits. What does this bitstring represent?

The binary numeral for n.

4. The C++ code below implements a function, "mystery." What does it compute?

```
float squre(float x)
 {
  return x*x;
 }

float mystery(float x, int k)
 {
  if (k == 0) return 1.0;
  else if(x == 0.0) return 0.0;
  else if (k < 0) return 1/mystery(x,-k);
  else if (k%2) return x*mystery(x,k-1);
  else return mystery(squre(x),k/2);
 }
```

It computes x^k.

5. Write the asymptotic time complexity for each code fragment, using Θ notation.

(a)
```
for (int i=1; i < n; i++)
    for (int j=i; j > 0; j--)
```
$\Theta(n^2)$

(b)
```
for (int i=1; i < n; i=2*i)
    for (int j=i; j < n; j++)
```
$\Theta(n \log n)$

(c)
```
for (int i=1; i < n; i = 2*i)
    for (int j=1; j < i; j++)
```
$\Theta(n)$

(d)
```
for (int i=1; i < n; i++)
    for (int j=1; j < i; j = j*2)
```
$\Theta(n \log n)$

(e)
```
for (int i=1; i < n; i++)
    for (int j=i; j < n; j = j*2)
```
$\Theta(n)$

(f)
```
for (int i=2; i < n; i = i*i)
```
$\Theta(\log \log n)$

(g)
```
for (int i=1; i*i < n; i++)
```
$\Theta(\sqrt{n})$

(h)
```
for (int i=n; i > 1; i = i/2)
    for (int j=1; j < i; j=2*j)
```
$\Theta(\log^2 n)$