University of Nevada, Las Vegas Computer Science 477/677 Spring 2023

Assignment 4: Due Saturday March 4, 2023, 11:59 PM

Name:____

You are permitted to work in groups, get help from others, read books, and use the internet. You will receive a message from the graduage assistant, Sepideh Farivar, telling you how to turn in the assignment.

1. Solve the following recurrences, expressing each answer using Θ notation.

   (a) $F(n) = F(n/3) + 1$

   $A = 1$, $B = 3$, $C = 0$, $A = B^C$.

   $F(n) = \Theta(\log n)$

   (b) $F(n) = F(2n/3) + 1$

   $A = 1$, $B = \frac{2}{3}$, $C = 0$, $A = B^C$.

   $F(n) = \Theta(\log n)$

   (c) $F(n) = F(2n/3) + F(n/3) + 1$

   $\alpha_1 = 1$, $\beta_1 = \frac{2}{3}$, $\alpha_2 = 1$, $\beta_2 = \frac{1}{3}$, $\gamma = 0$, and $\alpha_1\beta_1^\gamma + \alpha_2\beta_2^\gamma = 2 > 1$. Therefore we need to find δ such that $\alpha_1\beta_1^\delta + \alpha_2\beta_2^\delta = 1$. The choice is $\delta = 1$, which is larger than γ, Thus

   $F(n) = \Theta(n^\delta) = \Theta(n)$

   (d) $F(n) = F(2n/3) + F(n/3) + n$

   $\alpha_1 = 1$, $\beta_1 = \frac{2}{3}$, $\alpha_2 = 1$, $\beta_2 = \frac{1}{3}$, $\gamma = 1$, and $\alpha_1\beta_1^\gamma + \alpha_2\beta_2^\gamma = 1$. Therefore

   $F(n) = \Theta(n^\gamma \log n) = \Theta(n \log n)$

   (e) $F(n) = F(2n/3) + F(n/3) + n^2$

   $\alpha_1 = 1$, $\beta_1 = \frac{2}{3}$, $\alpha_2 = 1$, $\beta_2 = \frac{1}{3}$, $\gamma = 2$, and $\alpha_1\beta_1^\gamma + \alpha_2\beta_2^\gamma = \frac{5}{9} > 1$. Therefore

   $F(n) = \Theta(n^\gamma) = \Theta(n^2)$

   (f) $F(n) = F(12n/13) + F(5n/13) + 1$

   $\alpha_1 = 1$, $\beta_1 = \frac{12}{13}$, $\alpha_2 = 1$, $\beta_2 = \frac{5}{13}$, $\gamma = 0$, and $\alpha_1\beta_1^\gamma + \alpha_2\beta_2^\gamma = \frac{17}{13} > 1$. Letting $\delta = 2$, we have $\alpha_1\beta_1^\delta + \alpha_2\beta_2^\delta = \frac{144}{169} + \frac{25}{169} = 1$. Therefore

   $F(n) = \Theta(n^\delta) = \Theta(n^2)$

   (g) $F(n) = F(\log n) + 1$

   If you read the definition of $\log^*$ carefully, you can see that $\log^* n = 1 + \log^* \log n$. Thus

   $F(n) = \Theta(\log^* n)$

(h) $F(n) = F(\sqrt{n}) + 1$

Let $m = \log n$, and $G(m) = F(n)$. Then $F(\sqrt{n}) = G(\log \sqrt{n}) = G(\log n/2) = G(m/2)$ Thus $G(m) = G(m/2) + \log n)$. Solving that recurrence, we have $G(m) = \Theta(\log m)$. Thus

$F(n) = G(m) = \Theta(\log m) = \Theta(\log \log n)$

(i) $F(n) = 2F(n-1) + 1$

Let $n = \log m$. Let $F(n) = G(m) = G(2^n)$. Then $F(n-1) = G(2^{n-1}) = G(2^n/2) = G(m/2)$ The recurrence becomes $G(m) = 2G(m/2) + 1$. By the master theorem, $G(m) = \Theta(m)$. Thus

$F(n) = G(m) = \Theta(m) = \Theta(2^n)$

(j) $F(n) = 2F(\sqrt{n}) + \log n$

Let $m = \log n$, and $G(m) = F(n)$. $F(\sqrt{n}) = G(\log \sqrt{n}) = G(\log n/2) = G(m/2)$. The recurrence becomes $G(m) = 2G(m/2) + m$, thus

$F(n) = G(m) = \Theta(m \log m) = \Theta(\log n \log \log n)$.

(k) $F(n) = F(n/5) + F(7n/10) + n$

$\alpha_1 = 1$, $\beta_1 = \frac{1}{5}$ $\alpha_2 = 1$, $\beta_2 = \frac{7}{10}$, $\gamma = 1$, and $\alpha_1\beta_1^\gamma + \alpha_2\beta_2^\gamma = \frac{9}{10} < 1$. Thus

$F(n) = \Theta(n^\gamma) = \Theta(n)$

2. Name three very different applications of hashing.
Search structure.
Perfect hashing.
Security.

3. Phyllis, in her business in Southern California, used the last four digits of the customer's telephone number as a hash value for each customer.

(a) Would it have been better to use the first six digits of the customer's 10 digit phone number? Why or why not?

No. The area code and exchange would be correlated with the customer's location.

(b) Would it have been better to use the customer's 5 digit zip code?

No. The zip code would be correlated with the customer's location.

(c) Phyllis used separate chaining. She had no computer; all work was done by hand. The list of customers with each given 4-digit hash value was stored on an 8x5 file card. All 10000 cards were stored in a file cabinet. In your opinion, would it have been more efficient to use closed hashing?

No. Since all the work had to be done by hand, using a probe sequence would be very inconvenient.

4. SHA256 hashing is used to provide unique identifiers to data items. What is the probability that they are not really unique? Select one.

(a) **No.** Moderately large, but collisions are handled by secondary software.

(b) **No.** Small, but acceptable.

(c) **Yes.** The probability that a collision has ever happened since SHA256 was introduced is "astronomically" small.

5. Cuckoo hashing can let each item have three hash values instead of two. Here is an example with 8 data in a hash table ofsize 8. Can they be inserted into the table so that each item gets its own position? Show how that is done. When an item is ejected from a place, do not erase it; cross it out.

Ann	0	4	7
Bob	1	2	4
Cal	5	3	8
Dan	6	1	0
Eve	7	5	1
Fay	0	3	5
Gus	2	3	6
Hal	4	6	2

0	~~Ann~~ Fay
1	Bob
2	Gus
3	Cal
4	~~Ann~~ Hal
5	~~Cal~~ Eve
6	Dan
7	~~Eve~~ Ann