Assignment 1: Due Tuesday January 23, 2024

1. Problem 0.1 on page 8 of the textbook. Write either $O$, $\Omega$ or $\Theta$ in each blank. Write $\Theta$ if that is correct, otherwise write $O$ or $\Omega$.

(a) $n - 100 = \boxed{\Theta} (n - 200)$

(b) $n^{1/2} = \boxed{\Theta} (n^{2/3})$

(c) $100n + \log n = \boxed{\Theta} (n + \log^2 n)$

(d) $n \log n = \boxed{\Theta} (10n + \log(10n))$

(e) $\log(2n) = \boxed{\Theta} (\log(3n))$

(f) $10 \log n = \boxed{\Theta} (\log(n^2))$

(g) $n^{1.01} = \boxed{\Theta} (n \log^2 n)$

(h) $n^2 / \log n = \boxed{\Theta} (n \log^2 n)$

(i) $n^{0.1} = \boxed{\Theta} (\log^2 n)$

(j) $(\log n)^{\log n} = \boxed{\Theta} (n / \log n)$

(k) $\sqrt{n} = \boxed{\Theta} (\log^3 n)$

(l) $n^{1/2} = \boxed{\Theta} (5^{\log_2 n})$

(m) $n^{2^n} = \boxed{\Theta} (3^n)$

(n) $2^n = \boxed{\Theta} (2^{n+1})$

(o) $n! = \boxed{\Theta} (2^n)$

(p) $\log_2 n^{\log_2 n} = \boxed{\Theta} (2^{(\log_2 n)^2})$

(q) $\sum_{i=1}^{n} i^k = \boxed{\Theta} (n^{k+1})$
2. Look up Fibonacci numbers $F_1, F_2, F_3 \ldots$ if you are not familiar with them. Recall that $F_i + F_{i+1} = F_{i+2}$.

The first few Fibonacci numbers are 1, 1, 2, 3, 5, 8, …

Find the smallest constant $C$ such that $F_n = O(C^n)$.

3. Consider the following C++ program.

```cpp
void process(int n)
{
    cout << n << endl;
    if(n > 1) process(n/2);
    cout << n%2;
}

int main()
{
    int n;
    cout << "Enter a positive integer: ";
    cin >> n;
    assert(n > 0);
    process(n);
    cout << endl;
    return 1;
}
```

The last line of the output of `process(n)` is a string of bits. What does this bitstring represent?

4. The C++ code below implements a function, “mystery.” What does it compute?

```cpp
float squre(float x)
{
    return x*x;
}

float mystery(float x, int k)
{
    if (k == 0) return 1.0;
    else if(x == 0.0) return 0.0;
    else if (k < 0) return 1/mystery(x,-k);
    else if (k%2) return x*mystery(x,k-1);
    else return mystery(squre(x),k/2);
}
```