University of Nevada, Las Vegas Computer Science 477/677 Spring 2024

Answers to Assignment 5: Due Saturday March 30 2024

For each shortest path problem, let n be the number of vertices, and m the number of arcs.

1. What is the worst case asymptotic time complexity for each of these shortest path algorithms?
   (a) Bellman Ford. $O(nm)$
   (b) Floyd Warshall. $\Theta(n^3)$
   (c) Dijkstra. $O(m \log n)$
   (d) Johnson. $O(nm \log n)$
2. Write pseudocode for the Floyd Warshall algorithm. Let W[i,j] be the length of the arc from i to j, which could be ∞.

   The prorgrap computes $V[i,j]$, the length of the shortest path from i to j and the backpointer $B[i,j]$. $B[i,i]$ is undefined, since the shortest path from i itself has no arcs.

```
for(i = 1 to n)
 for(j = 1 to n)
   V[i,j] = W[i,j]
   B[i,j] = i
for i = 1 to n
  V[i,i] = 0

for j = 1 to n
  for i = 1 to n
    for k = 1 to n
      temp = V[i,j] + V[j,k]
      if(temp < V[i,k])
        V[i,k] = temp
        B[i,k] = B[j,k]
\begin{verbatim}

\item Write pseudocode for the Bellmain Ford algorithm. Be sure to include
the shortcut.
\vskip 1.0 in
The arcs are (s[k],t[k]) of weight W[k] for all k from 1 to m.
The source vertex is 0.
V[i] is the distance from 0 to i and B[i] is the backpointer of the shortest
path from 0 to i.

\begin{verbatim}
V[0] = 0
for i = 1 to n
```

```
  V[i] = infinity
finished = false
t = 0
while(not finished and t < n)
  {
   finished = true;
   t++
   for(k = 1 to m)
     i = s[k]
     j = t[k]
     temp = V[i] + W[k]
     if(temp < V[j])
       V[j] = temp
       B[j] = i
       finished = false
  }
```

The purpose of the variable t is to prevent the program from running forever if the graph contains a negative weight cycle.

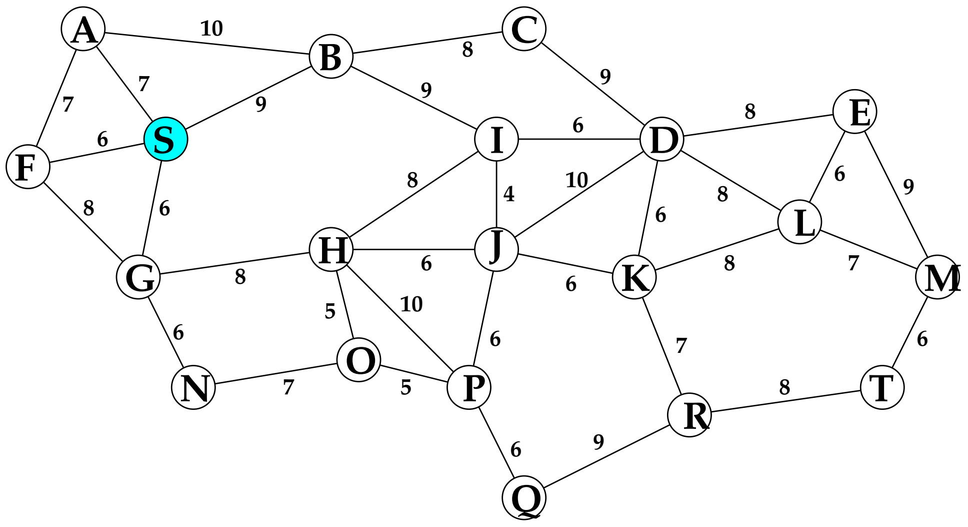
3. Walk through Dijkstra's algorithm for the single source minpath problem for the directed graph illustrated on the next page. Instead of numbering the vertices 0 through 19, I have assigned them letters from A to T. The source vertex is S.

   After each iteration of the main loop, show

   1. The array dist, where dist[x] is the smallest length of any path found so far from S to x. (Initially, dist[x] = ∞ for most x.)

   2. The array back, where back[x] is the next-to-the last vertex on the path of smallest weight found so far from S to x.

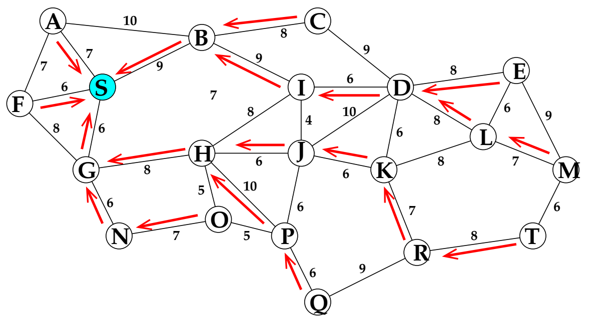
   3 The contents of heap. Do not try to show the structure of the heap, simply list its members.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	heap
dist																			0		S
back																			*		
dist	7	10				6	6												0		FGAB
back	S	S				S	S												*		
dist	7	10				6	6												0		GAB
back	S	S				S	S												*		
dist	7	10				6	6	14						12					0		ABNH
back	S	S				S	S	G						G					*		
dist	7	10				6	6	14						12					0		BNH
back	S	S				S	S	G						G					*		

Continue your work on Problem 3 on this page.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	heap
dist	7	10				6	6	14						12					0		BNH
back	S	S				S	S	G						G					*		
dist	7	10	17			6	6	14	18					12					0		NHCI
back	S	S	B			S	S	G	B					G					*		
dist	7	10	17			6	6	14	18					12	19				0		HCIO
back	S	S	B			S	S	G	B					G	N				*		
dist	7	10	17			6	6	14	18	20				12	19	24			0		CIOJP
back	S	S	B			S	S	G	B	H				G	N	H			*		
dist	7	10	17	26		6	6	14	18	20				12	19	24			0		IOJPD
back	S	S	B	C		S	S	G	B	H				G	N	H			*		
dist	7	10	17	26		6	6	14	18	20				12	19	24			0		OJPD
back	S	S	B	C		S	S	G	B	H				G	N	H			*		
dist	7	10	17	26		6	6	14	18	20				12	19	24			0		JPD
back	S	S	B	C		S	S	G	B	H				G	N	H			*		
dist	7	10	17	26		6	6	14	18	20	26			12	19	24			0		PDK
back	S	S	B	C		S	S	G	B	H	J			G	N	H			*		
dist	7	10	17	26		6	6	14	18	20	26			12	19	24	30		0		DKQ
back	S	S	B	C		S	S	G	B	H	J			G	N	H	P		*		
dist	7	10	17	26	34	6	6	14	18	20	26	34		12	19	24	30		0		KQEL
back	S	S	B	C	D	S	S	G	B	H	J	D		G	N	H	P		*		
dist	7	10	17	26	34	6	6	14	18	20	26	34		12	19	24	30	33	0		QREL
back	S	S	B	C	D	S	S	G	B	H	J	D		G	N	H	P	K	*		
dist	7	10	17	26	34	6	6	14	18	20	26	34		12	19	24	30	33	0		REL
back	S	S	B	C	D	S	S	G	B	H	J	D		G	N	H	P	K	*		
dist	7	10	17	26	34	6	6	14	18	20	26	34		12	19	24	30	33	0	41	ELT
back	S	S	B	C	D	S	S	G	B	H	J	D		G	N	H	P	K	*	R	
dist	7	10	17	26	34	6	6	14	18	20	26	34	41	12	19	24	30	33	0	41	TM
back	S	S	B	C	D	S	S	G	B	H	J	D	L	G	N	H	P	K	*	R	
dist	7	10	17	26	34	6	6	14	18	20	26	34	41	12	19	24	30	33	0	41	M
back	S	S	B	C	D	S	S	G	B	H	J	D	L	G	N	H	P	K	*	R	
dist	7	10	17	26	34	6	6	14	18	20	26	34	41	12	19	24	30	33	0	41	
back	S	S	B	C	D	S	S	G	B	H	J	D	L	G	N	H	P	K	*	R	



Backpointers are indicated.

4. Write C++ code (which must be able to be executed) for a dynamic program which finds the maximum weight legal subsequence of a sequence of positive integers. A subsequence is legal if it has no consecutive terms of the input sequence. Your program should use backpointers to recover the best subsequence.

   In order to make the grader's task easier, here are two sequences I want you to use while testing your program, each of length 20.

   15,324,184,48,102,31,119,15,26,160,129,78,14,7,64,83,22,88,185,123.

   139,79,19,53,52,23,94,298,209,59,16,116,80,130,279,195,6,37,161,206.

   Remember that the task of this assignment is not just to find the correct answers, but to write and run a correct program.

   I have given you two algorithms for this problem. Here is one of them: //

```
    #include<cstdio>
    #include<iomanip>
    #include<cassert>
    #include<string>
    #include<cmath>
    #include<iostream>
    #include<sstream>
    #include<stdio.h>
    #include<stdlib.h>
     using namespace std;

  const int n = 20; // number of coins
  const int sentinel = -1;
  int coin[n];
  int A[n]; // A[i] = maximum sum of legal subsequence ending at coin[i];
  int B[n];  // B[i] = backpointer

  void getcoins()
   {
    for(int i = 0; i < n; i++)
     cin >> coin[i];
   }

  void computeA()
   {
    A[0] = coin[0];
    B[0] = sentinel;
    A[1] = coin[1];
    B[1] = sentinel;
    A[2] = coin[0]+coin[2];
    B[2] = 0;
    for(int i = 3; i < n; i++)
    {
     int a1 = coin[i] + A[i-3];
     int a2 = coin[i] + A[i-2];
     //cout << "i = " << i << " a1 = " << a1 << " a2 = " << a2 << endl;
     if(a1 > a2)
      {
       A[i] = a1;
       B[i] = i-3;
      }
     else
      {
       A[i] = a2;
       B[i] = i-2;
      }
    }
  }

void writecoins()
 {
  for(int i = 0; i < n-1; i++)
   cout << coin[i] << ",";
  cout << coin[n-1] << "." << endl;
 }

void writebest(int n)
 {
  if(n > sentinel)
   {
    writebest(B[n]);
    cout << coin[n] << " + ";
   }
 }

void writebest()
 {
  if (A[n-1] > A[n])
   {
    writebest(B[n-1]);
    cout << coin[n-1] << " = " << A[n-1] << endl;
   }
  else
   {
    writebest(B[n]);
    cout << coin[n] << " = " << A[n-1] << endl;
```

```
      }
    }

   int main()
    {
     getcoins();
     writecoins();
     computeA();
     writebest();
     return 1;
    }
//
```

Here is the other //

```
     #include<cstdio>
     #include<iomanip>
     #include<cassert>
     #include<string>
     #include<cmath>
     #include<iostream>
     #include<sstream>
     #include<stdio.h>
     #include<stdlib.h>
      using namespace std;

   const int n = 20; // number of coins
   const int sentinel = -1;
   int coin[n];
   int A[n]; // A[i] = maximum sum of legal subsequence ending at coin[i];
   int B[n];  // B[i] = next to last coin of max legal subseq ending at coin[i];

   void getcoins()
    {
     for(int i = 0; i < n; i++)
      cin >> coin[i];
    }

   void computeA()
    {
     A[0] = coin[0];
     B[0] = sentinel;
     A[1] = coin[1];
     B[1] = sentinel;
     A[2] = coin[0]+coin[2];
```

```
  B[2] = 0;
  for(int i = 3; i < n; i++)
   {
    int a1 = coin[i] + A[i-3];
    int a2 = coin[i] + A[i-2];
    //cout << "i = " << i << " a1 = " << a1 << " a2 = " << a2 << endl;
    if(a1 > a2)
     {
      A[i] = a1;
      B[i] = i-3;
     }
    else
     {
      A[i] = a2;
      B[i] = i-2;
     }
   }
 }

void writecoins()
 {
  for(int i = 0; i < n-1; i++)
   cout << coin[i] << ",";
  cout << coin[n-1] << "." << endl;
 }

void writebest(int n)
 {
  if(n > sentinel)
   {
    writebest(B[n]);
    cout << coin[n] << " + ";
   }
 }

void writebest()
 {
  if (A[n-1] > A[n])
   {
    writebest(B[n-1]);
    cout << coin[n-1] << " = " << A[n-1] << endl;
   }
  else
   {
      writebest(B[n]);
      cout << coin[n] << " = " << A[n-1] << endl;
     }
   }

  int main()
   {
    getcoins();
    writecoins();
    computeA();
    writebest();
    return 1;
   }
//
```

However, the programs are equivalnet, that is, with the same input they give the same output.

Here are runs with the two input sequences I gave you.

```
39,79,19,53,52,23,94,298,209,59,16,116,80,130,279,195,6,37,161,206.
79 + 53 + 23 + 298 + 59 + 116 + 130 + 195 + 37 + 206 = 1196

15,324,184,48,102,31,119,15,26,160,129,78,14,7,64,83,22,88,185,123.
324 + 102 + 119 + 160 + 78 + 7 + 83 + 88 + 123 = 1084
```