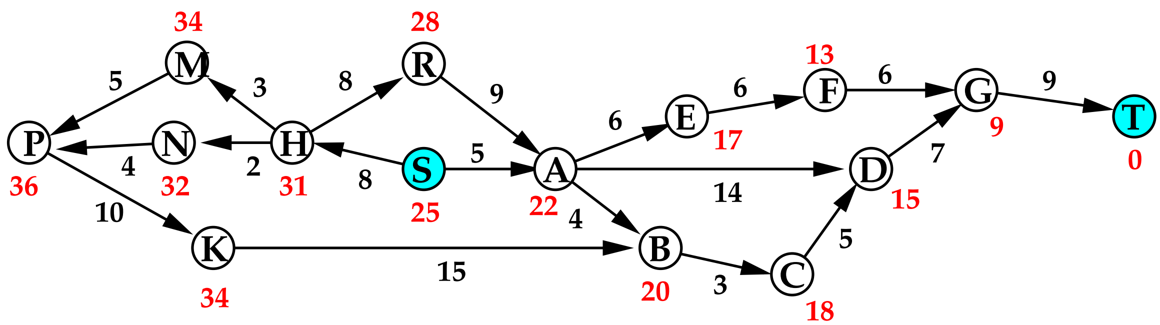
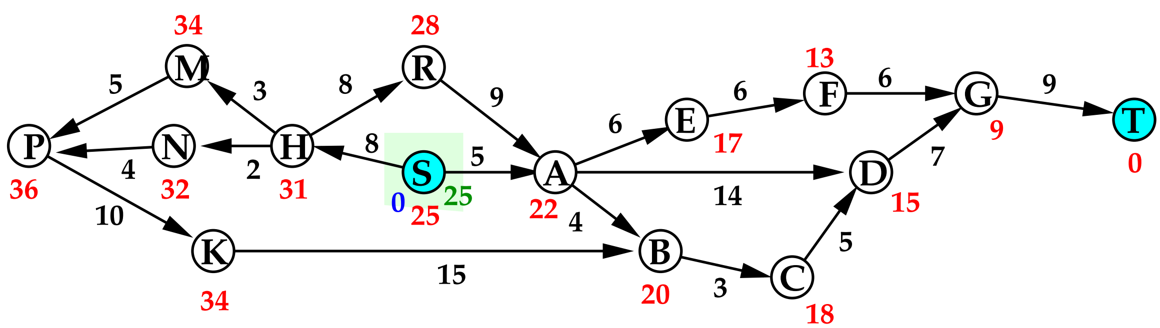
Corrected November 8, 2021

The A* Algorithm

We walk through an example computation of the A^* algorithm for solving the single pair minpath problem on a weighted directed graph. The pair is (S, T). Arc weights are shown as black numerals, we write $w(x, y)$ for the weight of the arc from x to y.



The heuristic $h(x)$ for each vertex x is indicated by a red numeral.



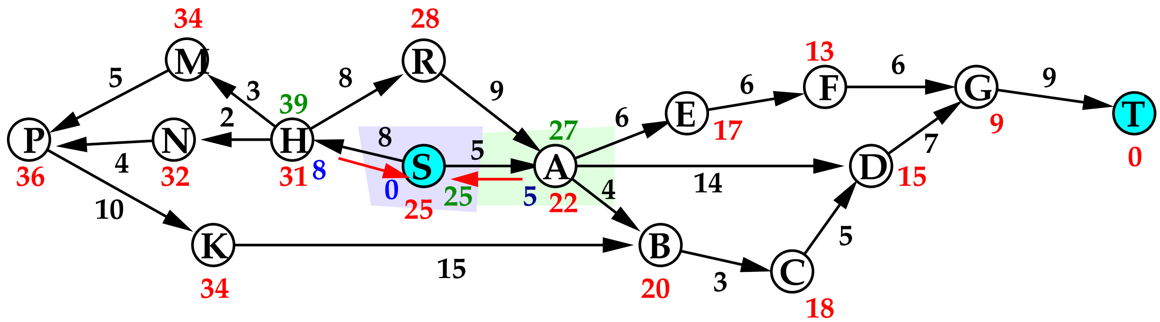
(a)

Just as for Dijkstra's algorithm, we maintain three sets of vertices: fully processed, indicated by a blue background, partially processed, indicated by a green background, and unprocessed, indicated by no background. The partially processed vertices are held in an updatable minqueue.

For each fully or partially processed vertex x, we let $f(x)$ be the length of the shortest path so far found, indicated by a blue numeral.

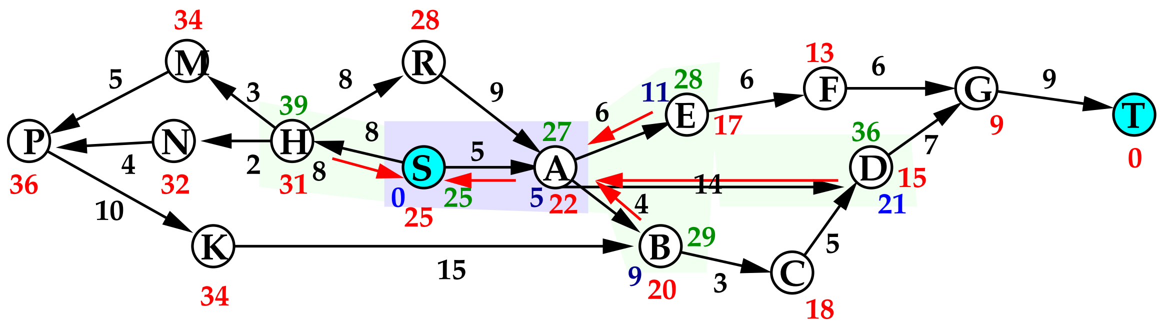
We let $g(x) = f(x) + g(x)$, indicated by a green numeral.

Initially, there are no fully processed vertices, and only the source vertex S is partially processed.



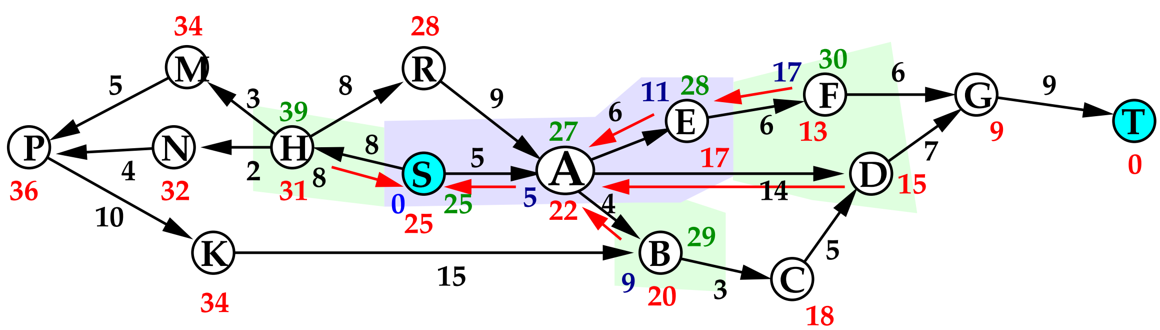
(b)

At each step, if $g(x)$ is the minimum value over all partially processed vertices, x becomes fully processed, and all its unprocessed out-neighbors become partially processed. During this step, S becomes fully processed, and A and H become partially processed.



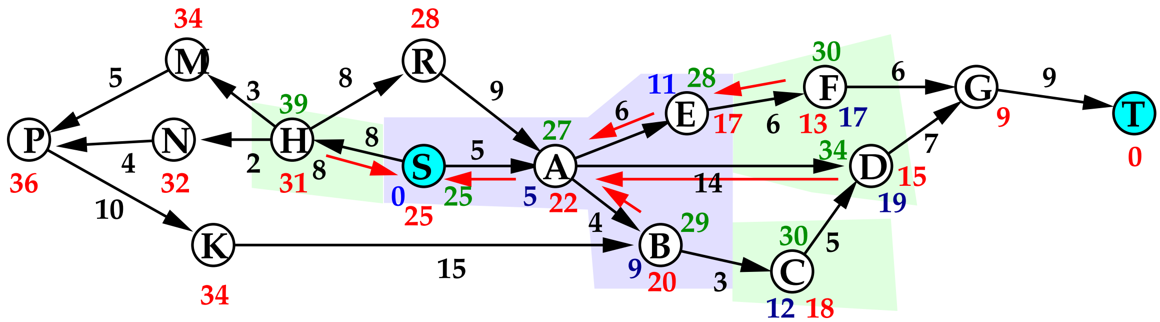
(c)

At this step, A becomes fully processed, while B, D, and E become partially processed. Backpointers are indicated as red arrows.



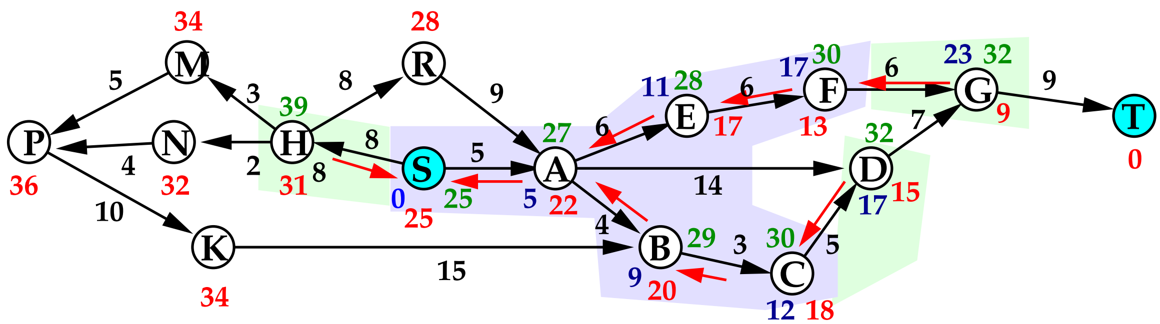
(d)

E becomes fully processed, while F becomes partially processed.



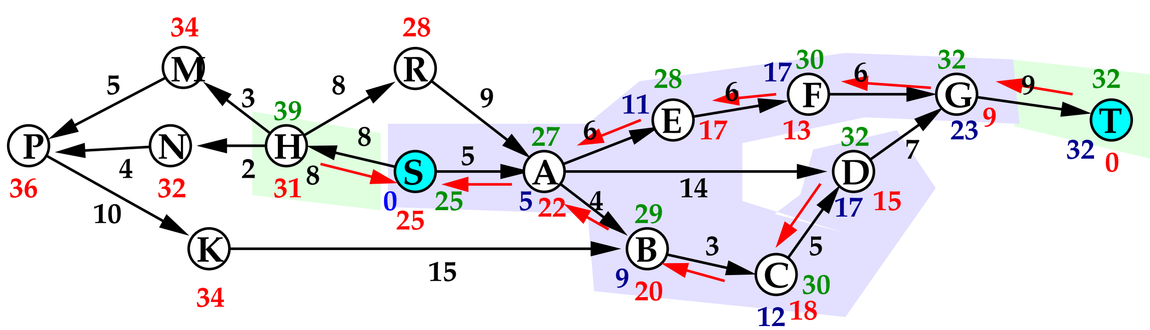
(e)

B becomes fully processed, while C becomes partially processed.



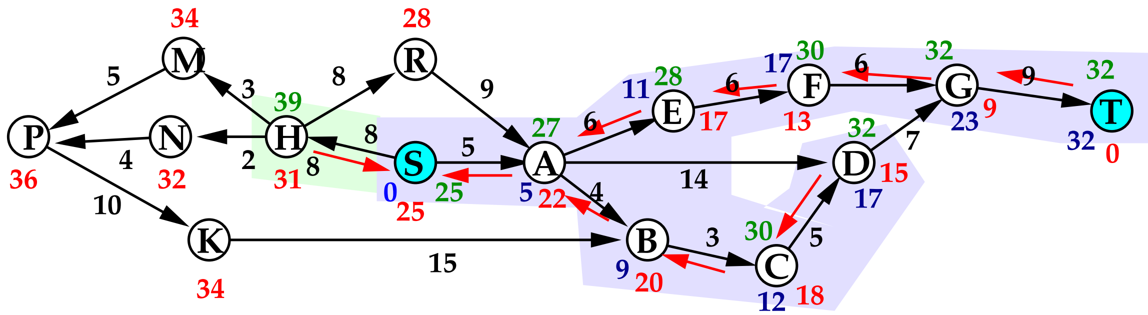
(f)

C becomes fully processed. D acquires a new, smaller value of f, and its backpointer changes to C.



(g)

D and G become fully processed, while T becomes partially processed.



(h)

It seems unnecessary, but the algorithm only stops when T becomes fully processed. Although not in this example, it is possible that T would acquire a new backpointer after being partially processed for the first time.

Errors fixed. If you detect another error, please send me email as soon as possible.