

Ordering Complexity Classes:
Problem 3–3 a, page 58
Cormen, Leiserson, Rivest, Stein: Second Edition

We write $f(n) \equiv g(n)$ to mean that $f(n) = \Theta(g(n))$, and we write $f(n) \ll g(n)$ to mean that $f(n) = O(g(n))$ but $g(n) \neq \Theta(g(n))$.

Constant:

$$1 \equiv n^{1/\lg n} = 2 \ll$$

I don't know a class name for these:

$$\lg(\lg^* n) \ll \lg^* n \equiv \lg^*(\lg n) \ll 2^{\lg^* n} \ll$$

I don't know a class name for these:

$$\ln \ln n \ll$$

Polylogarithmic:

$$\sqrt{\lg n} \ll \ln n \ll \lg^2 n \ll$$

Superpolylogarithmic and subpolynomial:

$$2^{\sqrt{2 \lg n}} \ll$$

Polynomial:

$$(\sqrt{2})^{\lg n} = \sqrt{n} \ll n = 2^{\lg n} \ll n \lg n \equiv \lg(n!) \ll n^2 = 4^{\lg n} \ll n^3 \ll$$

Superpolynomial and subexponential:

$$(\lg n)! \ll n^{\lg \lg n} = (\lg n)^{\lg n} \ll$$

Exponential:

$$\left(\frac{3}{2}\right)^n \ll 2^n \ll n \cdot 2^n \ll e^n \ll n! \ll (n+1)! \ll$$

Double Exponential:

$$2^{2^n} \ll 2^{2^{n+1}}$$