Dynamic Programming

3. For each of the following dynamic programming problems, what would you choose the subproblems to be?

   (a) Find the longest monotone subsequence of a sequence.

   The length of the longest monotone subsequence of the i^{th} prefix of the sequence which ends at the i^{th} term of the sequence, for all i up to the length of the sequence.

   (b) Find the edit distance between two strings.

   The edit distance between the i^{th} prefix of the first string and the j^{th} prefix of the second string, for each i up to the length of the first string and each j up to the length of the second string.

   (c) Find the longest common subsequence of two sequences.

   The longest common subsequence of the i^{th} prefix of the first sequence and the j^{th} prefix of the second sequence, for each i up to the length of the first sequence and each j up to the length of the second sequence.

   (d) Find the shortest distance between vertices s and t in a weighted acyclic directed graph.

   The shortest distance between s and and x, for each vertex x of the graph.

   (e) The knapsack problem.

   We assume that the input numbers are integers. If $x_1, \ldots x_n$ are the items and S is the size of the knapsack, then the knapsack problem for items $x_1, \ldots x_i$ and knapsack j, for each $i \leq n$ and each $j \leq S$.

   (f) Find the n^{th} Fibonacci number.

   The i^{th} Fibonacci number, for all $i \leq n$.