Computer Science 302 Fall 2009 (Practice) First Examination, September 17, 2009

Name:____

No books, notes, or scratch paper. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire examination is 305 points.

1. True or False. [5 points each]

   (a) ______ $n = O(n^2)$

   (b) ______ $n^2 = O(n)$

   (c) ______ Mergesort sorts n items in $O(n \log n)$ steps.

   (d) ______ $n^{0.001} = O(\log n)$

   (e) ______ Bubblesort sorts n items in $O(n \log n)$ steps.

2. Fill in the blanks.

   (a) [10 points] We say that an algorithm uses *divide and conquer* if the algorithm first breaks the problem up into smaller problems, solves each of the smaller problems recursively, and then combines those answers to answer the original problem. We have studied two divide and conquer sorting algorithms so far this semester; namely ____________________ and ____________________.

   (b) [10 points] Any comparison-based sorting algorithm on a list of n items uses at least ____________________ comparisons in the worst case.

3. Using "Big O" notation state how many times "Hello world" will be printed for each of the pseudo-code fragments below, in terms of n. Give the best possible answer. For example, if you write $O(n^2)$ as the answer to (a), technically that is correct but it will be marked wrong because $O(n)$ is a better answer.

   (a) [0 points]

   ```
   for(int i=1; i<n; i++)
     cout << "Hello world" << endl;
   ```

   (b) [10 points]

   ```
   for(int i=1; i<n; i++)
     for(int j=1; j<n; j++)
       cout << "Hello world" << endl;
   ```

   (c) [10 points]

   ```
   for(int i=1; i<n; i++)
     for(int j=i; j<n; j++)
       cout << "Hello world" << endl;
   ```

   (d) [10 points]

   ```
   for(int i=n; i>1; i=i/2)
     for(int j=1; j<i; j++)
       cout << "Hello world" << endl;
   ```

   (e) [10 points]

   ```
   for(int i=1; i<n; i++)
     for(int j=1; j<n; j=2*j)
       cout << "Hello world" << endl;
   ```

   (f) [10 points]

   ```
   for(int i=1; i<n; i++)
     for(int j=1; j<i; j=2*j)
       cout << "Hello world" << endl;
   ```

   (g) [10 points]

   ```
   for(int i=1; i<n; i++)
     for(int j=i; j<n; j=2*j)
       cout << "Hello world" << endl;
   ```

   (h) [10 points] The following problem is a bit harder than the others.

   ```
   for(int i=1; i*i<n; i++)
     cout << "Hello world" << endl;
   ```

   (i) [10 points] The following problem is a lot harder than the others.

   ```
   for(int i=1; i<n; i=2*i)
     for(int j=1; j<=i; j=2*j)
       cout << "Hello world" << endl;
   ```

4. [20 points]

   Order the following functions by asymptotic class. Be sure to indicate which functions are in the same asymptotic class.

   n, n^2, $\log n$, $n \log n$, $n \log\log n$, $\log(n^2)$, $\log^2 n$, $\sqrt{n}$, $\frac{(2n+3)^4}{(n+2)^3}$, $n!$, $\log(n!)$, 2^n, n^n, 3^n, $5n + \sqrt{n} + \log n + 15$

5. [30 points] In the programming assignment, you must have written some code for sorting an array using selection sort. Write that code here, using the same variable names you used in your assignment.

   Do not attempt to write the entire assignment; just the selection sort portion. You should only write a few lines.

6. [30 points] According to the master theorem, if $F(n) = AF(n/B) + n^C$ then

$$F(n) = \begin{cases} & \text{if} \\ & \text{if} \\ & \text{if} \end{cases}$$

7. [30 points] Using the master theorem, give a solution to each of the following recurrences.

   (a) $F(n) = 2F(n/2) + n$

   (b) $F(n) = 2F(n/3) + n$

   (c) $F(n) = 4F(n/2) + n$

8. [30 points] Walk through the algorithm which uses a stack to evaluate a postfix expression, where the input is the expression 9 4 * 2 7 + - and you show the stack using an array implementation.

9. [20 points] Suppose that the items of a queue are A, H, K, B, T in that order, where A is the front item.

   (a) Sketch the appearance of a circular linked list implementation of that queue.

   (b) Insert the item L into that queue. Show the steps. (You should draw at least two additional figures.)

10. [20 points] Write down the steps of *polyphase mergesort*, where the initial list is (G,H,E,W,R,I,V,C,M,Q,T). You should simply show the intermediate lists and the final sorted list. Additional explanations are unnecessary.