

Name: \_\_\_\_\_

This practice examination is much longer than the real final examination will be. If you can work all the problems here, you will probably be well-prepared for the final examination.

This part of the practice examination is 235 points.

15. True or False. [5 points each]

- (a) \_\_\_\_\_ A binary search tree is commonly used to represent unfulfilled obligations.
- (b) \_\_\_\_\_ Quicksort takes  $O(n \log n)$  time to sort an array of  $n$  items.
- (c) \_\_\_\_\_ Given the choice between two algorithms, one of which takes  $O(n)$  time and the other of which takes  $O(n^2)$  time, it is *always* best to choose the one which takes  $O(n)$  time.
- (d) \_\_\_\_\_ The built-in function `random` is an excellent choice for a hash function.

16. [5 points] What are the smallest and largest possible number of nodes that a 2-3 tree with height 3 could have? \_\_\_\_\_ and \_\_\_\_\_.

17. [20 points] Walk through the steps of the stack algorithm used to evaluate the following postfix expression, showing the stack at each step: (Hint: there will be 9 illustrations of the stack.)

5 6 + 3 \* 2 3 \* -

18. [20 points] Find an optimal prefix code for the alphabet  $\{A, B, C, D, E, F, G, H\}$ , if the frequencies of the symbols are as given in the following table:
- |     |    |
|-----|----|
| $A$ | 35 |
| $B$ | 7  |
| $C$ | 32 |
| $D$ | 5  |
| $E$ | 16 |
| $F$ | 4  |
| $G$ | 11 |
| $H$ | 5  |
19. [30 points]
- The *Partition* step of Quicksort has a loop invariant. Give that loop invariant, and illustrate its meaning by drawing a figure, or figures.
20. [30 points] Explain the difference between dynamic programming and memoization.

21. [20 points] Draw “before” and “after” figures illustrating *left rotation*.

22. [30 points] If you use *Heapsort* to sort an array, the first step is to *heapify* the array. Given the following array, show the steps of that heapification, assuming that your final array will be sorted in alphabetical order from left to right. The number of rows in the table below may or may not be the correct number of steps; you might not use all the rows, or you might have to add more rows.

<i>A</i>	<i>L</i>	<i>G</i>	<i>O</i>	<i>R</i>	<i>I</i>	<i>T</i>	<i>H</i>	<i>M</i>

23. [30 points] Suppose you are writing a dynamic programming algorithm to find the minimum weight path between a given source vertex  $S$  and a given target vertex  $T$  in a weighted directed acyclic graph  $G$ .
- (a) Describe the subproblems.
  - (b) In what order would you work the subproblems?

24. [30 points] Use tropical (min-plus) matrix multiplication to solve the all-pairs minimum weight path problem for the weighted directed graph shown in Figure 1.

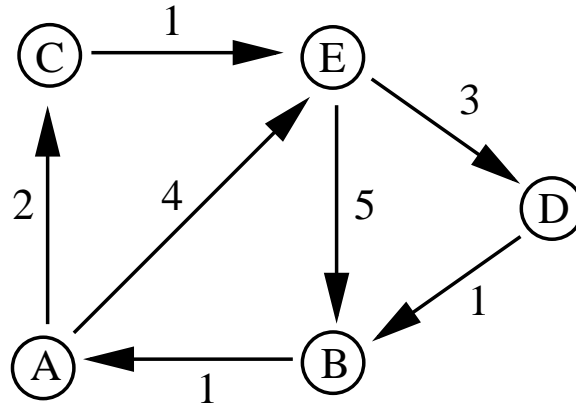


Figure 1: A weighted directed graph for problem 24

235. Points.