University of Nevada, Las Vegas Computer Science 302 Spring 2018

Assignment 9: Due April 23, 2018

The Collatz Conjecture. For any positive integer n, let $f(n) = \begin{cases} n/2 \text{ if } n \text{ is even} \\ 3n+1 \text{ if } n \text{ is odd} \end{cases}$. The conjecture is that, if we start with any positive integer, and repeatedly apply f, we will eventually reach 1. For example, the Collatz sequence of 5 is 5, 16, 8, 4, 2, 1, while the Collatz sequence of 9 is 9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

We define $f^*(n)$ to be the number of times we need to apply f to reach 1, starting from n. Thus, for example, $f^*(5) = 5$, and $f^*(9) = 19$. If we never reach 1, we define $f^*(n) = \infty$. However, in all known cases, $f^*(n)$ is finite.

Your project is to write a program that builds a sparse array, implemented using a search structure, which holds the values of $f^*(n)$ for all n up to 10, and for all other numbers which appear in the Collatz sequences of those numbers. If you used an ordinary array, it would have to have size 52 for $N = 10$, 9232 for $N = 100$, and 250504 for $N = 1000$. What a waste of space!

The function $f^*(n)$, also called the **total stopping time** of n, can be defined by the following recurrence:

$$f^*(n) = \begin{cases} 0 \text{ if } n = 1 \\ 1 + f^*(f(n)) \text{ otherwise} \end{cases}$$

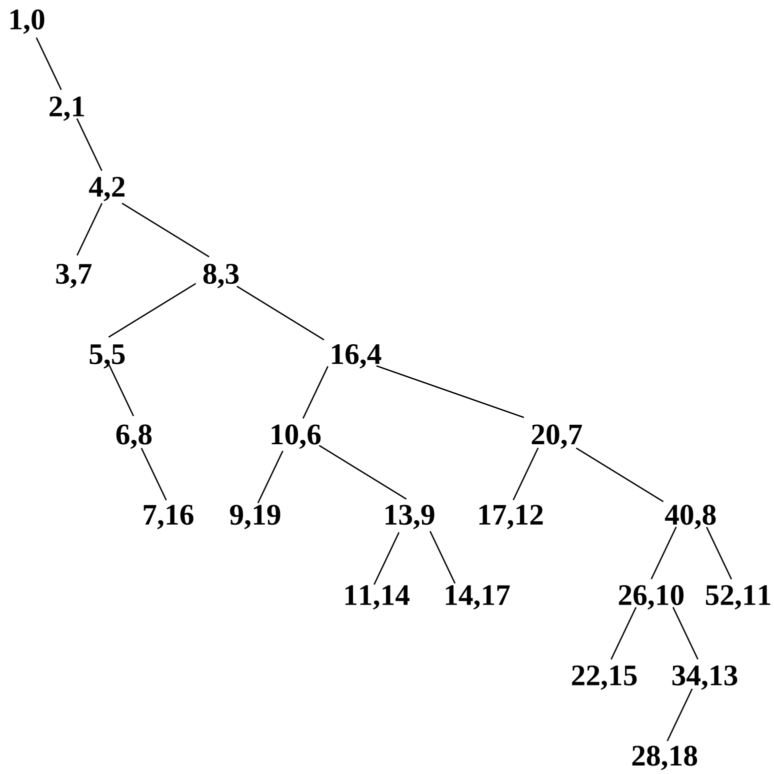
Your search structure holds ordered pairs of the $(n, f^*(n))$, where n is the key. Fetch(n) is executed by searching the structure. When you find the pair (n, V), you know that $f^*(n) = V$.

Do the following steps. For the first run, $N = 10$.

1. Initialize your search structure as empty, and then insert the pair $(1, 0)$, since $f^*(1) = 0$.
2. If you execute fetch(n) for any n, the value of $f^*(n)$ might already have been computed, in which case the pair $(n, f^*(n))$ will already be in the search structure.
3. If $f^*(n)$ has not been computed, you must first calculate it, then insert it. You first compute $m = f(n)$. You then obtain the value of $f^*(m)$ by executing fetch. You then let $f^*(n) = 1 + f^*(n)$, then insert the pair $(n, f^*(n)$ into the search structure.
4. Of course, $f^*(m)$ might not have already been computed, in which case you must recursively execute the steps 2. and 3. for m.
5. Your output consists of the list of values of $f^*(n)$ for $n = 1, 2, \ldots N$. I also printed a message each time a new memo was inserted into the search structure, other than the initial memo.
6. Run the program for $N = 10$ and then for $N = 100$. Print out the number of entries in the search structure.

Refer to the Wikipedia page: `https://en.wikipedia.org/wiki/Collatz_conjecture` There are other pages on the internet that deal with the same problem.

I am not telling you what to use for a search structure, but I used a binary search tree. Here is my binary search tree for $N = 10$. I did not try to maintain balance, so the tree looks rather lopsided.



My output for $N = 10$ is as follows.

```
f*(1) = 0
inserting f*(2) = 1
f*(2) = 1
inserting f*(4) = 2
inserting f*(8) = 3
inserting f*(16) = 4
inserting f*(5) = 5
inserting f*(10) = 6
inserting f*(3) = 7
f*(3) = 7
f*(4) = 2
f*(5) = 5
inserting f*(6) = 8
f*(6) = 8
inserting f*(20) = 7
inserting f*(40) = 8
inserting f*(13) = 9
inserting f*(26) = 10
inserting f*(52) = 11
inserting f*(17) = 12
inserting f*(34) = 13
inserting f*(11) = 14
inserting f*(22) = 15
inserting f*(7) = 16
f*(7) = 16
f*(8) = 3
inserting f*(14) = 17
inserting f*(28) = 18
inserting f*(9) = 19
f*(9) = 19
f*(10) = 6
There are 22 memos  Tree height = 9
```