Additional Complexity Problems Name _____

For each piece of code, assume that a positive integer **n** has been given. Give the asymptotic complexity of each in terms of **n**, using Θ notation. Most of these problems are hard, some are even harder, and some are even harder than that.

1. It is sometimes useful to include a *counter* into your code to help measure complexity. The counter must be incremented once in the body of each subprogram (including the main program) and once inside each loop. The incrementation in a loop should not be inside any nested loop. Here is an example:

```
int kount = 1;
for (int i = 0; i < n; i++)
 {
  kount++;
  for j = 0, j < n; j++
   {
    kount++;
    cout << "kount = " << kount << endl;
   }
 }
```

   Note that the incrementation for the outer loop is not inside the inner loop. What is the asymptotic time complexity of this fragment of code?

   Answer: $\Theta(n^2)$.

2. The next two loops have the same time complexity.

   (a) `for (int i = n; i > 1; i = i/2)`

   (b) `for (int i = 1; i < n; i = 2*i)`

   Answer: Both of these are $\Theta(\log n)$.

3. This problem is just a bit harder.

   Note: you do not need braces around the body of a loop if that body consists of a single statement.

```
for (int i = 1; i < n; i = 2*i)
 for (int j = i; j > 0; j = j-1)
  cout << "i = " << i << " j - " << j << end;
```

4. The next one is hard.

```
for (int i = 0; i*i < n; i++)
```

5. This one is much harder, in my opinion.

```
for (int i = 2; i < n; i = i*i)
```

Name _____

6. It gets worse. Your code contains the procedure **george** declared as follows:

```
void george(int i)
 {
  if (i > 0)
   {
    george(i-1);
    george(i-1);
   }
 }
```

   and then, somewhere else in the code, you have the following statement:

```
george(n);
```

   What is the time complexity of that statement? Hint: you would *never* want to write this procedure in real life.

7. The next problem is similar to the previous one, but it has an answer that might surprise you.

```
void martha(int i)
 {
  if (i > 1)
   {
    martha(i/2);
    martha(i/2);
   }
 }
```

   What is the time complexity of the following statement, which is somewhere else in the code?

```
martha(n);
```

8. Finally, perhaps the hardest of all:

```
for (int i = 2; i < n; i = i*i)
 for (int j = i; j > 1; j = j/2)
```