Computer Science 456/656 Fall 2018 Practice Examination October 17, 2018

The entire examination is 445 points.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (a) _____ Every subset of a regular language is regular.

   (b) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m, n \geq 0$. Then L is a regular language.

   (c) _____ The complement of every regular language is regular.

   (d) _____ The Kleene closure of every context-free language is context-free.

   (e) _____ If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

   (f) _____ If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

   (g) _____ There is a PDA that accepts all valid C++ programs.

   (h) _____ The intersection of any two regular languages is regular.

   (i) _____ The language consisting of all base 7 numerals for positive integers n such that $n \ \% \ 3 = 2$ is regular.

   (j) _____ The intersection of any two context-free languages is context-free.

   (k) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n c^m$, where $m, n \geq 0$. Then L is a context-free language.

   (l) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then L is a context-free language.

   (m) _____ The complement of every context-free language is context-free.

   (n) _____ The union of any two context-free languages is context-free.

   (o) _____ If a language has an context-free grammar, then it is is accepted by some push-down automaton.

   (p) _____ Every context-free language has an unambiguous context-free grammar.

   (q) _____ Every deterministic machine is a non-deterministic machine.

   (r) _____ The language consisting of all base 2 numerals for integer powers of 2 is regular.

   (s) _____ There is a DPDA that accepts the language of all palindromes over the binary alphabet $\{0, 1\}$.

   (t) _____ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (u) _____ The language $\left\{a^i b^j c^k \mid j \geq i + k\right\}$ is context-free.

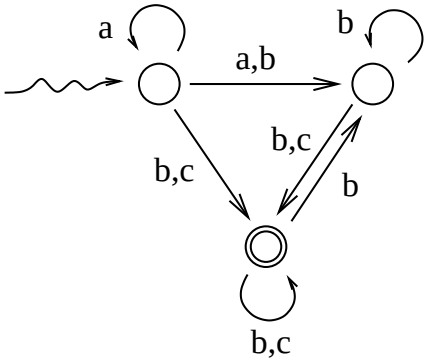
2. [10 points] Fill in the blanks. An LALR _____ outputs a _____ derivation.

3. [20 points] State the pumping lemma for regular languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

4. [20 points] State the pumping lemma for context-free languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

5. [25 points] Draw an NFA with five states which accepts the language described by the regular expression $(0+1)^*0(0+1)(0+1)(0+1)$

6. [25 points] Write a regular expression for the language accepted by the following NFA. If your answer is unnecessarily long by a wide margin, I might mark it wrong even if it's right.



Find a Regular Expression

7. [20 points] Let G be the context-free grammar given below.

$S \to a$

$S \to wS$

$S \to iS$

$S \to iSeS$

Prove that G is ambiguous by writing two different **leftmost** derivations for the string *iwiaea*. [If you simply show two different parse trees, you are not following instructions.]

8. [30 points] Design a PDA that accepts the language $L = \{a^n b c^n \ : \ n \geq 0\}$.

9. [30 points] Give a context-free grammar for the language of all strings over $\{0, 1\}$ of the form $0^m 1^n$ where $n \neq m$.

10. [30 points] The following context-free grammar G is ambiguous. Give an equivalent unambiguous grammar.

1. $E \ \to E \ + \ E$
2. $E \ \to E \ - \ E$
3. $E \ \to E \ * \ E$
4. $E \ \to - \ E$
5. $E \ \to (\ E \ )$
6. $E \ \to a$
7. $E \ \to b$
8. $E \ \to c$

11. Fill in the ACTION and GOTO tables for the ambiguous grammar given in Problem 10 (This problem is hard. I will give more clues in class.)

	a	b	c	$+$	$-$	$*$	(	)	**eof**	E
0										
1									**halt**	
2										
3										
4										
5										
6										
7										
8										
9										
10										
11										
12										
13										
14										
15										

12. Let L be the language generated by the context-free grammar

    1. $S \to a$
    2. $S \to wS$
    3. $S \to iS$
    4. $S \to iSeS$

    (a) [20 points] Give a CNF grammar which generates L.

    (b) [20 points] Is your CNF grammar ambiguous? If so, prove it is ambigous by giving two different parse trees for *iwiaea* using that CNF grammar.

13. [30 points] Consider the NFA whose transition diagram is in Figure 1 below. where the input alphabet is $\{a, b, c\}$. Draw the transition diagram of an equivalent minimal DFA. Show your steps.

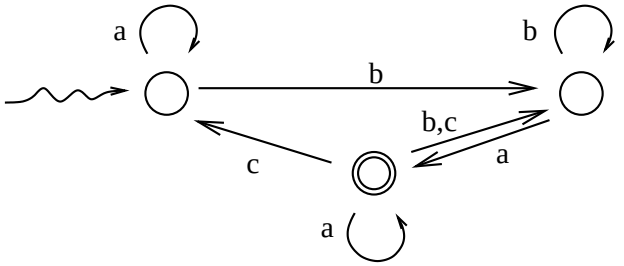


Figure 1: Find a minimal DFA equivalent to this NFA

14. [30 points] Let $L = \left\{w \in \{a, b\}^* \mid \#_a(w) = 2\#_b(w)\right\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w. For example, $aaababaaabba \in L$, because that string has the twice as many a's as b's. Give a context-free grammar for L. Your grammar may be ambiguous.

15. [30 points]

    1. $S \to \varepsilon$
    2. $S \to a_2 S_3 b_4 S_5$

	a	b	**eof**	S
0				
1			**halt**	
2				
3				
4				
5				

Complete the ACTION and GOTO tables of an LALR parser for the grammar given above. This grammar unambiguously generates the Dyck language, where (to avoid difficulty grading) a represents a left parenthesis, and b represents a right parenthesis. Example strings include ε, ab, $aabb$, $abab$, and $aabbab$.