University of Nevada, Las Vegas Computer Science 456/656 Fall 2024

Practice Problems for the Final Examination on December 11, 2024

Don't forget the True/False problem.

1. State the pumping lemma for regular languages.

2. State the pumping lemma for context-free languages.

3. Prove that every decidable language is enumerated in canonical order by some machine.

4. Prove that every language that is enumerated in canonical order by some machine is decided by some other machine.

5. Prove that any language accepted by any machine can be enumerated by some other machine.

6. Prove that any language which is enumerated by some machine is accepted by some other machine.

7. The Dyck language is generated by the following context-free grammar. (As usual, to make grading easier, I use a and b for left and right parentheses.)

   1. $S \to aSbS$
   2. $S \to \lambda$

   Use the pumping lemma to prove that the Dyck language is not regular.

8. Every language, or problem, falls into exactly one of these categories. For each of the languages, write a letter indicating the correct category.

A Known to be $\mathcal{NC}$.
B Known to be $\mathcal{P}$–TIME, but not known to be $\mathcal{NC}$.
C Known to be $\mathcal{NP}$, but not known to be $\mathcal{P}$–TIME and not known to be $\mathcal{NP}$–complete.
D Known to be $\mathcal{NP}$–complete.
E Known to be $\mathcal{P}$–SPACE but not known to be $\mathcal{NP}$
F Known to be EXP–TIME but not nown to be $\mathcal{P}$–SPACE.
G Known to be EXP–SPACE but not nown to be EXP–TIME.
H Known to be decidable, but not nown to be EXP–SPACE.
K $\mathcal{RE}$ but not decidable.
L co-$\mathcal{RE}$ but not decidable.
M Neither $\mathcal{RE}$ nor co-$\mathcal{RE}$.

(i) ________ SAT.

(ii) ________ 3-SAT.

(iii) ________ 2-SAT.

(iv) ________ The Independent Set problem.

(v) ________ The Subset Sum Problem.

(vi) ________ All positions of RUSH HOUR from which it is possible to win.

(vii) ________ All C++ programs which do not halt if given themselves as input.

(viii) ________ All base 10 numerals for perfect squares.

(ix) ________ The Dyck language.

(x) ________ The Jigsaw problem. (That is, given a finite set of two-dimensional pieces, can they be assembled into a rectangle, with no overlap and no spaces.)

(xi) ________ Factorization of binary numerals.

(xii) ________ All C++ programs which halt with no input.

(xiii) ________ The Boolean Circuit Problem.

(xiv) ________ All configurations of RUSH HOUR from which it's possible to win.

(xv) ________ All pairs $(\langle M \rangle, w)$ such that M is a machine which halts with input w.

(xvi) ________ All positions of generalized checkers (any size board) from which Black can force a win.

(xvii) ________ Does a square matrix with integer entries have determinan zero?

(xviii) ________ All satisfiable Boolean expressions.

(xix) ________ All binary numerals for composite integers. (Composite means not prime.)

(xx) ________ All binary numerals for multiples of 3

(xxi) ________ All binary numerals for square integers, that is $0, 1, 100, 1001, \ldots$

(xxii) ________ All pairs of binary numerals $(\langle p \rangle, \langle q \rangle)$ such that p has a divisor greater than q.

(xxiii) ________ All pairs of context-free grammars $(\langle G_1 \rangle, \langle G_2 \rangle)$ such that $L(G_1) = L(G_2)$.

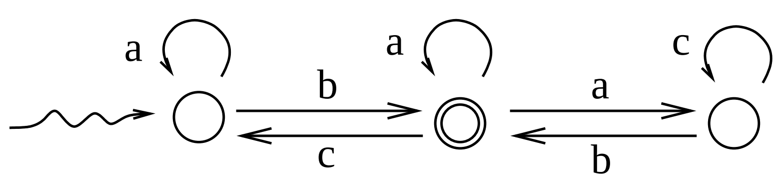
9. Give a definition of each term.

   (a) Accept. (That is, what does it mean for a machine to accept a language.)

   (b) Decide. (That is, what does it mean for a machine to decide a language.)

   (c) Canonical order of a language L.

10. Give a regular expression for the language accepted by the machine in the figure below.



11. Which class of languages does each of these machine classes accept?

    (a) Deterministic finite automata. ________________________________

    (b) Non-deterministic finite automata. ________________________________

    (c) Push-down automata. ________________________________

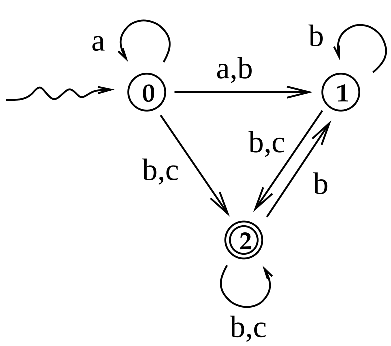
    (d) Limited push-down automata. (LPDA) ________________________________

    An LPDA is a PDA with a limit on its stack size. That is, there is some integer D such that the stack can hold no more than D symbols.

    (e) Turing Machines. ________________________________

12. Let L be the binary language which consists of all strings which contain the substring 111. Construct a DFA which accepts L.

13. Construct a minimal DFA equivalent to the NFA shown below.



14. The grammar below is an ambiguous CF grammar and is parsed by the LALR parser whose Action and Goto tables are shown. Write a computation of the parser for the input string *iiwaea*.

1. $S \to i_2 S_3$
2. $S \to i_2 S_3 e_4 S_5$
3. $S \to w_6 S_7$
4. $S \to a_8$

	ACTION					GOTO
	a	i	e	w	\$	S
0	$s8$	$s2$		$s6$		1
1					halt	
2	$s8$	$s2$		$s6$		3
3			$s4$		$r1$	
4	$s8$	$s2$		$s6$		5
5			$r2$		$r2$	
6	$s8$	$s2$		$s6$		7
7			$r3$		$r3$	
8			$r4$		$r4$	

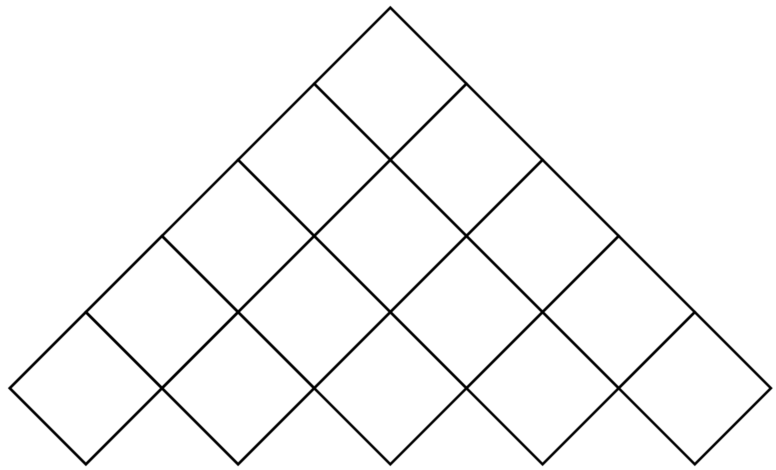
15. Find an NFA which accepts the language generated by this grammar.

$S \to aA|cS|cC$
$A \to aA|bS|cB|\lambda$
$B \to aA|cB|bC|\lambda$
$C \to aB$

16. Use the CYK algorithm to decide whether *abcab* is generated by the CNF grammar:

$S \to AB\,|\,BC\,|CA$
$A \to a$
$B \to SA\,|\,SS\,|\,b$
$C \to c$

by filling in the matrix.



17. Consider the grammar G below.
1. $E \to E -_2 E_3$
2. $E \to E *_4 E_5$
3. $E \to x_6$

Prove that G is ambiguous by giving two different parse trees for $x - x * x$.
One of those parse trees is the "right" one, meaning that it respects the standard precedence of operators.
The LALR parser given below will not give the "right" parse tree. Indicate the changes you need to make to correct this problem.

	ACTION				GOTO
	x	$-$	$*$	$\$$	E
0	$s6$				1
1		$s2$	$s4$	*halt*	
2	$s6$				3
3		$r1$	$r1$	$r1$	
4	$s6$				5
5		$s2$	$s4$	$r2$	
6		$r3$	$r3$	$r3$	

18. Give the verifier-certificate definition of the class $\mathcal{NP}$.

19. What is the importance nowadays of $\mathcal{NC}$?

20. What complexity class contains sliding block problems?

21. Give a polynomial time reduction of the 3-SAT to the independent set problem.

22. The grammar below is an ambiguous CF grammar with start symbol E, and is parsed by the LALR parser whose Action and Goto tables are shown here. The Action table is missing actions for the second column, when the next input symbol is the "minus" sign. Fill it in. Remember the C++ precedence of operators. (Hint: the column has seven different actions: s2, s4, r1, r2, r3, r4, and r5, some more than once, and has no blank spaces.)

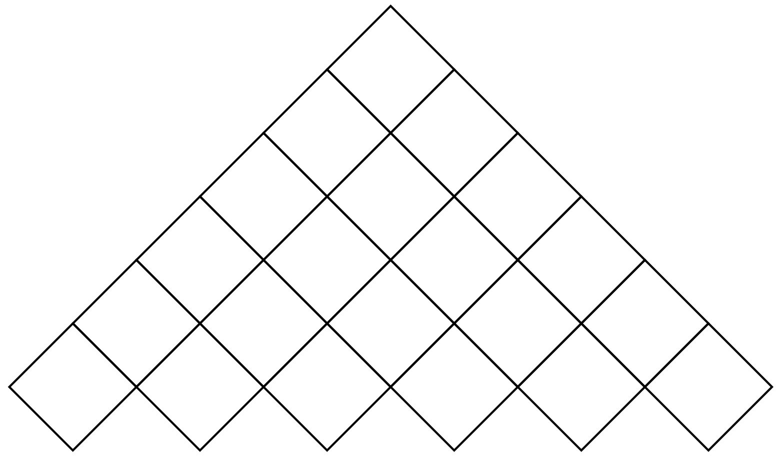
1. $E \to E -_2 E_3$
2. $E \to -_4 E_5$
3. $E \to E *_6 E_7$
4. $E \to (_8 E_9)_{10}$
5. $E \to x_{11}$

	ACTION						GOTO
	x	$-$	$*$	(	)	\$	S
0	$s11$			$s8$			1
1			$s6$			halt	
2	$s11$			$s8$			3
3			$s6$		$r1$	$r1$	
4	$s11$			$s8$			5
5			$r2$		$r2$	$r2$	
6	$s11$			$s8$			7
7			$r3$		$r3$	$r3$	
8	$s11$			$s8$			9
9			$s6$		$s6$		
10			$r4$	$r4$	$r4$	$r4$	
11			$r5$		$r5$	$r5$	

23. Let $L = \{w \in \{a,b\}^* : \#_a(w) = \#_b(w)\}$, that is, each string of L has equal numbers of each symbol. Draw a DPDA which accepts L.

24. Use the CYK algorithm to decide whether $x - x - -x$ is generated by the CNF grammar below, by filling in the matrix.

$E \to ME$
$A \to EM$
$E \to AE$
$M \to -$
$E \to x$



25. Consider the CF grammar below. The ACTION and GOTO tables are given below, except that six actions are mising, indicated by question marks. Fill in the missing actions (below the question marks). The actions of your table must be consistent with the precedence of operators in C++.

1. $E \to E -_2 E_3$
2. $E \to -_4 E_5$
3. $E \to E *_6 E_7$
4. $E \to x_8$

	ACTION				GOTO
	x	$-$	$*$	$\$$	E
0	$s8$	$s4$			1
1		$s2$	$s6$	HALT	
2	$s8$	$s4$			3
3		?	?	$r1$	
4	$s8$	$s4$			5
5		?	?	$r2$	
6	$s8$	$s4$			7
7		?	?	$r3$	
8	$s8$	$r4$	$r4$	$r4$	

26. Give a polynomial time reduction of the subset sum problem to partition.

27. Prove that the halting problem is undecidable.

29. Label each of the following sets as countable or uncountable.

(a) ______________________ The set of integers.

(b) ______________________ The set of rational numbers.

(c) ______________________ The set of real numbers.

(d) ______________________ The set of binary languages.

(e) ______________________ The set of co-$\mathcal{RE}$ binary languages.

(f) ______________________ The set of undecidable binary languages.

(g) ______________________ The set of functions from integers to integers.

(h) ______________________ The set of recursive real numbers.

30. What is the Church-Turing Thesis? Why is it important?