University of Nevada, Las Vegas Computer Science 456/656 Fall 2025

Assignment 3: Due Saturday October 4, 2025 at 23:59:59

You are permitted to work in groups, get help from others, read books, and use the internet.

$\mathcal{P}$ means $\mathcal{P}$–TIME.

1. True/False. If the answer is not known to science at this time, enter "O" for Open.

   (i) _____ Every subset of a regular language is regular.

   (ii) _____ co-$\mathcal{P} = \mathcal{P}$.

   (iii) _____ co-$\mathcal{NP} = \mathcal{NP}$.

   (iv) _____ co-$\mathcal{P}$–SPACE = $\mathcal{P}$–SPACE.

   (v) _____ Block placement problems are $\mathcal{NP}$.

   (vi) _____ Sliding block problems are $\mathcal{P}$–SPACE.

   (vii) _____ $\mathcal{P}$–SPACE = $\mathcal{NP}$

   (viii) _____ Regular expression equivalence is decidable.

   (ix) _____ Context-free grammar equivalence is decidable.

   (x) _____ Every context-free language is context-sensitive.

   (xi) _____ The language C++ is context-free.

   (xii) _____ The intersection of any two context-free languages is context-free.

   (xiii) Let $L = \{a^n b^n c^n \ : \ n \geq 0\}$.

      i. _____ L is context-free.

      ii. _____ The complement of L is context-free.

   (xiv) _____ Every language is countable.

   (xv) _____ The set of real numbers is countable.

   (xvi) _____ The set of all C++ programs is countably infinite.

   (xvii) _____ Every CF language is generated by a CNF grammar.

   (xviii) _____ For any real number x, there is a program that prints the decimal expansion of x.

   (xix) _____ For any real number x, there is a machine that decides whether a given rational number is less than x.

   (xx) _____ There are only countably many decidable binary languages. (A binary language is defined to be any language over the binary alphabet $\{0, 1\}$.)

   (xxi) _____ Given an NFA with n variables that accepts L, there exists a regular grammar G with n variables that generates L.

   (xxii) _____ $\left\{a^i b^j c^k \ : \ i = k\right\}$ is a context-free language.

   (xxiii) _____ Given a binary numeral $\langle n \rangle$ it is possible to find the prime factors of n in time which is polynomial in $|\langle n \rangle|$.

(xxiv) _____ Given a binary numeral $\langle n \rangle$ it is possible to decide whether n is prime in time which is polynomial in $|\langle n \rangle|$.

(xxv) _____ Any language generated by a context-sensitive grammar is decidable.

(xxvi) _____ The complement of any decidable language is decidable.

(xxvii) _____ The union of any two decidable languages is decidable.

(xxviii) _____ The complement of any undecidable language is undecidable.

(xxix) _____ The union of any two undecidable languages is undecidable.

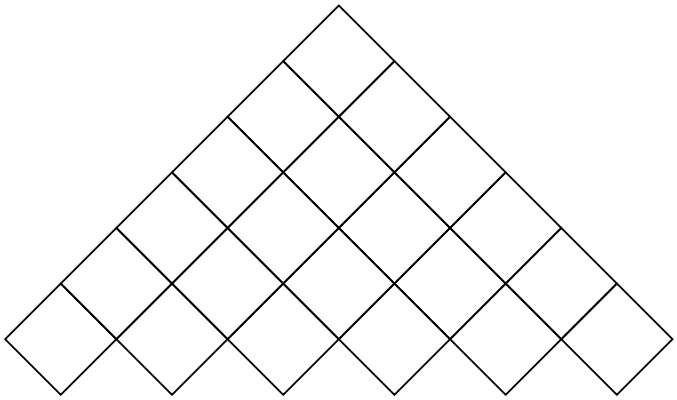
(xxx) _____ Every context-free language is accepted by some PDA.

(xxxi) _____ Every language generated by an unambiguous CF language is accepted by some DPDA.

2. Let L be the language generated by the following CNF (Chomsky Normal Form) grammar.

   $S \to IS$
   $S \to AB$
   $A \to IS$
   $B \to ES$
   $S \to WS$
   $S \to a$
   $I \to i$
   $E \to e$
   $W \to w$

   Use the CYK algorithm and the table shown to prove that $iwiaea \in L$.



3. The context free grammar G given in Problem 2 is ambiguous, and generates the string $w = iwiaea$ in two different ways.

   (a) Using G, write two different left-most derivations of w.

   (b) Using G, write two different right-most derivations of w.

   (c) Using G, draw two different parse trees for w.

4. List the grammar classes and language classes of the Chomsky hierarchy.

5. Give two context-free languages whose intersection is not context-free.

6. Write a grammar for the Dyck language (using 'a' and 'b' instead of parentheses) and give a derivation of the string abaabb.

7. Sketch a PDA which accepts $L = \{w \in \{a, b\}^* : \#_a(w) > \#_b(w)\}$, that is, strings which have more a's than b's.

8. State the pumping lemma for regular languages. The quantifiers and conditionals must be properly positioned within the statement of the lemma. If your answer has all the right words in the wrong order, you have not answered the question correctly.

9. In the following, do not write more than necessary. Your answers should be concise and correct.

   (a) What could be a certificate to prove that a given Boolean expression E is in the language SAT?

   (b) Explain the verification definition of the class $\mathcal{NP}$.

10. Read this Wikipedia page: `https://en.wikipedia.org/wiki/NP-completeness`