

University of Nevada, Las Vegas Computer Science 456/656 Fall 2025

Assignment 4: Due October 18, 2025, 11:59:59 PM

Name: _____

You are permitted to work in groups, get help from others, read books, and use the internet. You will receive a message from the graduate assistant, Sabrina Wallace, telling you how to turn in the assignment.

$\mathcal{P}$ means $\mathcal{P}$ -TIME.

1. True/False. If the answer is not known to science at this time, enter "O" for Open.
 - (a) _____ There are only countably many decidable binary languages.
 - (b) _____ If a number x can be approximated to any desired accuracy by a computer, then x is a recursive real number.
 - (c) _____ If some machine can compute an increasing sequence of fractions which converges to x , then x must be a recursive real number. (The sequence can be infinite.)
2. State the pumping lemma for regular languages. If your answer contains all the right words, but not in the right order, you might get no credit.
3. Given languages L_1 and L_2 , exactly one of the following statements is correct. Which one? _____
 - (i) If there is an easy reduction from L_1 to L_2 and L_1 is hard, then L_2 must be hard.
 - (ii) If there is an easy reduction from L_1 to L_2 and L_2 is hard, then L_1 must be hard.
 - (iii) If there is an easy reduction from L_1 to L_2 and L_1 is easy, then L_2 must be easy.
 - (iv) If there is a hard reduction from L_1 to L_2 and L_2 is easy, then L_1 must be hard.

4. Give a polynomial time reduction of subset sum to the partition problem.

5. Give a polynomial time reduction of 3-SAT to the independent set problem.

6. Prove that the halting problem is undecidable.

For the following two problems, either take a course in music theory, or read the webpage <https://www.math.uwaterloo.ca/~mrubinst/tuning/12.html>.

7. Prove that $\log_2 3$ is irrational.

8. $\log_2 3$ is irrational, but is very close to the rational number $19/12$, only about 1% off. Explain why this fact is important for Western music.¹

¹From the internet: “Western music may be defined as organized instrumentation and sound created and produced in Europe, the United States, and other societies established and shaped by European immigrants. This includes a wide assortment of musical genres, from classical music and jazz to rock and roll and country-western music.”