University of Nevada, Las Vegas Computer Science 456/656 Fall 2025

Assignment 3: Due Saturday October 4, 2025 at 23:59:59

You are permitted to work in groups, get help from others, read books, and use the internet.

$\mathcal{P}$ means $\mathcal{P}$–TIME.

1. True/False. If the answer is not known to science at this time, enter "O" for Open.

   (i) **F** Every subset of a regular language is regular.

   (ii) **T** co-$\mathcal{P} = \mathcal{P}$.

   (iii) **O** co-$\mathcal{NP} = \mathcal{NP}$.

   (iv) **T** co-$\mathcal{P}$–SPACE $=$ $\mathcal{P}$–SPACE.

   (v) **T** Block placement problems are $\mathcal{NP}$.

   (vi) **T** Sliding block problems are $\mathcal{P}$–SPACE.

   (vii) **O** $\mathcal{P}$–SPACE $= \mathcal{NP}$

   (viii) **T** Regular expression equivalence is decidable.

   (ix) **F** Context-free grammar equivalence is decidable.

   (x) **T** Every context-free language is context-sensitive.

   (xi) **F** The language C++ is context-free.

   (xii) **F** The intersection of any two context-free languages is context-free.

   (xiii) Let $L = \{a^n b^n c^n \ : \ n \geq 0\}$.

      i. **F** L is context-free.

      ii. **T** The complement of L is context-free.

   (xiv) **T** Every language is countable.

   (xv) **F** The set of real numbers is countable.

   (xvi) **T** The set of all C++ programs is countably infinite.

   (xvii) **T** Every CF language is generated by a CNF grammar.

   (xviii) **F** For any real number x, there is a program that prints the decimal expansion of x.

   (xix) **F** For any real number x, there is a machine that decides whether a given rational number is less than x.

   (xx) **T** There are only countably many decidable binary languages. (A binary language is defined to be any language over the binary alphabet $\{0, 1\}$.)

   (xxi) **T** Given an NFA with n variables that accepts L, there exists a regular grammar G with n variables that generates L.

   (xxii) **T** $\left\{a^i b^j c^k \ : \ i = k\right\}$ is a context-free language.

   (xxiii) **O** Given a binary numeral $\langle n \rangle$ it is possible to find the prime factors of n in time which is polynomial in $|\langle n \rangle|$.

(xxiv) **T** Given a binary numeral $\langle n \rangle$ it is possible to decide whether n is prime in time which is polynomial in $|\langle n \rangle|$.

(xxv) **T** Any language generated by a context-sensitive grammar is decidable.

(xxvi) **T** The complement of any decidable language is decidable.

(xxvii) **T** The union of any two decidable languages is decidable.

(xxviii) **T** The complement of any undecidable language is undecidable.

(xxix) **F** The union of any two undecidable languages is undecidable.

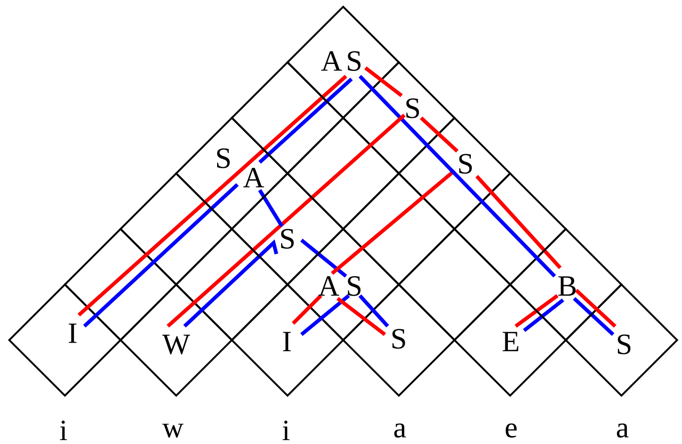
(xxx) **T** Every context-free language is accepted by some PDA.

(xxxi) **F** Every language generated by an unambiguous CF language is accepted by some DPDA.

2. Let L be the language generated by the following CNF (Chomsky Normal Form) grammar.

$S \to IS$
$S \to AB$
$A \to IS$
$B \to ES$
$S \to WS$
$S \to a$
$I \to i$
$E \to e$
$W \to w$

Use the CYK algorithm and the table shown to prove that $iwiaea \in L$.



3. The context free grammar G given in Problem 2 is ambiguous, and generates the string $w = iwiaea$ in two different ways.

(a) Using G, write two different left-most derivations of w.

$S \Rightarrow IS \Rightarrow iS \Rightarrow iWS \Rightarrow iwS \Rightarrow iwAB \Rightarrow iwISB \Rightarrow$
$iwiSB \Rightarrow iwiaB \Rightarrow iwiaES \Rightarrow iwiaeS \Rightarrow iwiaea$

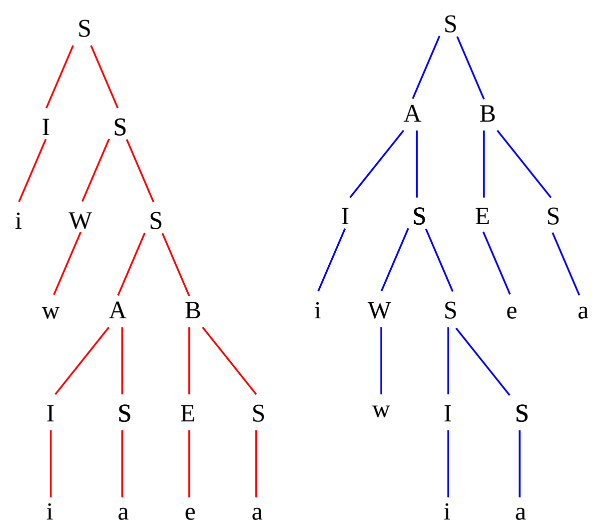
$S \Rightarrow AB \Rightarrow ISB \Rightarrow iSB \Rightarrow iWSB \Rightarrow iwSB \Rightarrow iwISB \Rightarrow$
$iwiSB \Rightarrow iwiaB \Rightarrow iwiaES \Rightarrow iwiaeS \Rightarrow iwiaea$

(b) Using G, write two different right-most derivations of w.

$S \Rightarrow IS \Rightarrow IWS \Rightarrow IWAB \Rightarrow IWAES \Rightarrow IWAEa \Rightarrow IWAea \Rightarrow$
$IWISea \Rightarrow IWIaea \Rightarrow IWiaea \Rightarrow Iwiaea \Rightarrow iwiaea$

$S \Rightarrow AB \Rightarrow AES \Rightarrow AEa \Rightarrow Aea \Rightarrow ISea \Rightarrow IWSea \Rightarrow IWISea$
$IWIaea \Rightarrow IWiaea \Rightarrow Iwiaea \Rightarrow iwiaea$

(c) Using G, draw two different parse trees for w.



4. List the grammar classes and language classes of the Chomsky hierarchy.

   Type-0, general (or unrestricted) grammars, recursively enumerable languages.

   Type-1, context-sensitive grammars, context-sensitive languages.

   Type-2, context-free grammars, context-free languages.

   Type-3, regular grammars, regular languages.

5. Give two context-free languages whose intersection is not context-free.

   There are many correct answers. Here is one I gave in class:

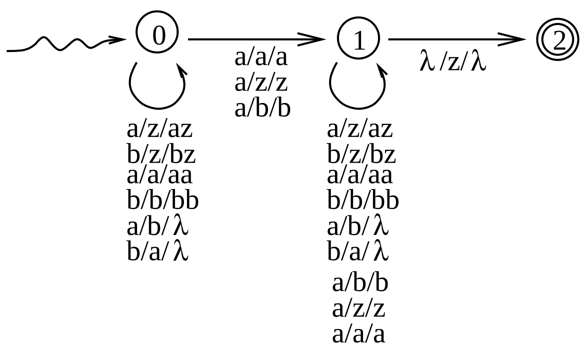
   $L_1 = \left\{a^n b^n c^k \ : \ n \geq 0, k \geq 0\right\}$ $L_2 = \left\{a^k b^n c^n \ : \ n \geq 0, k \geq 0\right\}$

6. Write a grammar for the Dyck language (using 'a' and 'b' instead of parentheses) and give a derivation of the string abaabb.

   $S \to aSbS$
   $S \to \lambda$

   Here is the unique left-most derivation:
   $S \Rightarrow aSbS \Rightarrow abS \Rightarrow abaSbS \Rightarrow abaaSbSbS \Rightarrow abaabSbS \Rightarrow abaabbS \Rightarrow abaabb$

7. Sketch a PDA which accepts $L = \left\{w \in \{a, b\}^* \ : \ \#_a(w) > \#_b(w)\right\}$, that is, strings which have more a's than b's.



   The key is that the machine must read and discard at least one a. All other a's and b's must be matched. In state 0 and state 1, any number of a's and b's are matched and any number of additional a's are discarded. But in the arc from 0 to 1, exactly one a is read and discarded. The arc to 2 guarantees that the stack is empty.

8. State the pumping lemma for regular languages. The quantifiers and conditionals must be properly positioned within the statement of the lemma. If your answer has all the right words in the wrong order, you have not answered the question correctly.

   For any regular language L,
   There is a positive integer p such that
   For any string $w \in L$ of length at least p,
   There are strings x, y, z such that the following statements hold:
   1. $w = xyz$
   2. $|xy| \leq p$
   3. $|y| \geq 1$ (that is, y is not the empty string)
   4. **For any integer** $i \geq 0$, $xy^i z \in L$.

   Note the nesting of the five quantifiers.

9. In the following, do not write more than necessary. Your answers should be concise and correct.

   (a) What could be a certificate to prove that a given Boolean expression E is in the language SAT?

   The clear choice is that a certificate is a satisfying assignment of E.

   (b) Explain the verification definition of the class $\mathcal{NP}$.

   A language L if $\mathcal{NP}$ if and only if there exists a machine V (we call this the verifier) such that:

   i. For any $w \in L$, there exists a string c (called a *certificate* of w) such that V accepts the pair w, c in time which is polynomial in the length of w.
   ii. For any $w \notin L$ and any string c, V does **not** accept the pair w, c.

   Do not forget that the second condition is just as important as the first.

   A guide string is one form of a certificate. Typically, w is an instance of a problem and c is a solution to that instance. For example, for the jigsaw puzzle problem, an instance is a box of pieces, and a certificate is a completed puzzle.

10. Read this Wikipedia page: `https://en.wikipedia.org/wiki/NP-completeness`

    You don't have to give an answer to this. But be sure you open the page! If you don't get it, read it to the end anyway, then perhaps again. You will have learned something, even if you think you haven't.