

University of Nevada, Las Vegas Computer Science 456/656 Fall 2025

Assignment 4: Due October 11, 2025, 11:59:59 PM

Name: _____

You are permitted to work in groups, get help from others, read books, and use the internet. You will receive a message from the graduate assistant, Sabrina Wallace, telling you how to turn in the assignment.

$\mathcal{P}$ means $\mathcal{P}$ -TIME.

1. True/False. If the answer is not known to science at this time, enter “O” for Open.
 - (a) **T** There are only countably many decidable binary languages.
 - (b) **T** If a number x can be approximated to any desired accuracy by a computer, then x is a recursive real number.
 - (c) **F** If some machine can compute an increasing sequence of fractions which converges to x , then x must be a recursive real number. (The sequence can be infinite.)
2. State the pumping lemma for regular languages. If your answer contains all the right words, but not in the right order, you might get no credit.

Notice the five nested quantifiers. Mixing them up means you haven’t answered the question. For any regular language L , there is a positive integer p such that for any $w \in L$ of length at least p , there exist strings x, y, z such that the following four statements hold.

1. $w = xyz$
 2. $|xy| \leq k$
 3. y is not the empty string
 4. For any integer $i \geq 0$, $xy^iz \in L$.
3. Given languages L_1 and L_2 , exactly one of the following statements is correct. Which one? **(i)**
 - (i) If there is an easy reduction from L_1 to L_2 and L_1 is hard, then L_2 must be hard.
 - (ii) If there is an easy reduction from L_1 to L_2 and L_2 is hard, then L_1 must be hard.
 - (iii) If there is an easy reduction from L_1 to L_2 and L_1 is easy, then L_2 must be easy.
 - (iv) If there is a hard reduction from L_1 to L_2 and L_2 is easy, then L_1 must be hard.
 4. Give a polynomial time reduction of subset sum to the partition problem.

Given an instance $I_K = (x_1, x_2, \dots, x_m, K)$ of Subset Sum, let $S = \sum_{i=1}^m x_i$. Let $R(I_K) = I_P$ be the following instance of Partition:

$$I_P = (x_1, \dots, x_m, K + 1, S - K + 1)$$

That is, $I_P = (y_1, \dots, y_\ell)$ where $\ell = m + 2$, $y_i = x_i$ for $i \leq m$, $y_{m+1} = K + 1$, and $y_{m+2} = S - K + 1$.

5. Give a polynomial time reduction of 3-SAT to the independent set problem. A figure might help, but is not required.

Let $E = C_1 \cdot C_2 \cdot \dots \cdot C_K$ be a Boolean expression in 3-CNF form. We can assume each clause has exactly three terms, since we can pad a clause with duplicate terms. For example, we can replace $x + y$ with $x + x + y$, and x with $x + x + x$.

Let $C_i = t_{i,1} + t_{i,2} + t_{i,3}$, the disjunction of three terms. Thus, E contains $3K$ terms.

Define a graph G_E whose vertices are $\{x_{i,j} : 1 \leq i \leq K, j = 1, 2, 3\}$. That is there is a 1-1 correspondence between terms of E and vertices of G_E . The vertices $v_{i,1}, v_{i,2}, v_{i,3}$ form a 3-clique, a complete subgraph of G_E . Besides the clique edges, G_E has contradiction edges, namely an edge from any $v_{i,j}$ to $v_{i',j'}$ if $t_{i,j}$ contradicts $t_{i',j'}$. E is satisfiable if and only if G_E has an independent set of K vertices.

6. Prove that the halting problem is undecidable.

Theorem 1 *HALT is not decidable.*

Proof: By contradiction. Suppose HALT is decidable. Let D be a machine which implements the following program:

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read a machine description  $\langle M \rangle$ .
if  $M$  halts with input  $\langle M \rangle$ 
    run forever.
else
    halt.

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We now run D with input $\langle D \rangle$. One of the following two cases must hold.

Case 1. D halts with input $\langle D \rangle$. That means that, when D reads $\langle D \rangle$, it runs forever, hence D does not halt with input $\langle D \rangle$, contradiction.

Case 2. D does not halt with input $\langle D \rangle$. That means that, when D reads $\langle D \rangle$, it halts, hence D halts with input $\langle D \rangle$, contradiction.

In either case, we obtain a contradiction, hence HALT is undecidable. ■

For the following two problems, either take a course in music theory, or read the webpage <https://www.math.uwaterloo.ca/~mrubinst/tuning/12.html>.

7. Prove that $\log_2 3$ is irrational.

Assume that $\log_2 3$ is equal to a fraction, p/q . Then $2^{p/q} = 3$. Take both sides to the power of q . Then $2^p = 3^q$. 2^p must be even, while 3^q must be odd, contradiction. Thus, $\log_2 3$ is irrational.

8. $\log_2 3$ is irrational, but is very close to the rational number $19/12$, only about 0.1% off. Explain why this fact is important for Western music.¹

$\log_2 3 \approx 1.584962501$, is close to $19/12 \approx 1.583333333$. Thus $2^{19} = 524288$ is close to $3^{12} = 531441$. The most important harmonic in music is the $3/2$ ratio of the frequencies of C and G, the so-called

¹From the internet: "Western music may be defined as organized instrumentation and sound created and produced in Europe, the United States, and other societies established and shaped by European immigrants. This includes a wide assortment of musical genres, from classical music and jazz to rock and roll and country-western music."

“perfect fifth.” If we continue increasing by this ratio, we obtain the sequence of notes: C, G, D, A, E, B, F $\sharp$, D $\flat$, A $\flat$, E $\flat$, B $\flat$, F, C, rising seven octaves, which should multiply the frequency by 2^7 . But this cannot be achieved, since $(3/2)^{12}$ is not exactly 2^7 , although it’s close. To correct for this discrepancy, the scale must be “tempered,” meaning some of the frequency ratios in the sequence must be adusted slightly so as to end with a note exactly seven octaves higher.