University of Nevada, Las Vegas Computer Science 456/656 Fall 2025

Answers to Assignment 7: Due November 19, 2025

Name:____

There are problems in this homework that were on the second examination. I have concentrated on problems which many students did badly. They will likely be repeated in the next examination or the final examination.

$\mathcal{P}$ means $\mathcal{P}$–TIME.

1. True/False. If the answer is not known to science at this time, enter "O" for Open.

   (a) **T** Every context-sensitive language is decidable.

   (b) **F** If a convergent sequence of rational numbers can be written by some machine, its limit must be a recursive real number.

   (c) **F** If G is a context-free grammar, there must be an LALR parser which constructs a parse tree for any $w \in L(G)$.

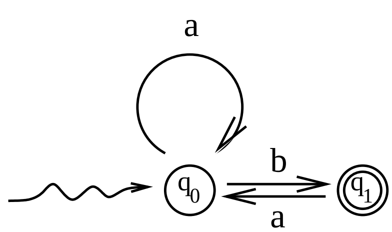
2. Give an example of an $\mathcal{NC}$ language which is not context-free.

   All strings of the form a^n such that n is a power of 2.

   This language can easily be proved not regular by the pumping lemma, and any context-free language over an alphabet of size 1 is regular. There also a direct proof using the pumping lemma for context-free languages.

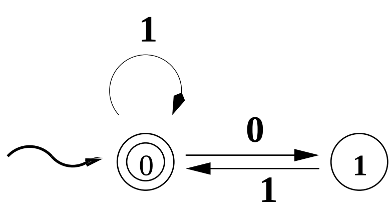
3. Write a regular expression for each of these regular languages.

   (a) The language accepted by the machine illustrated below.



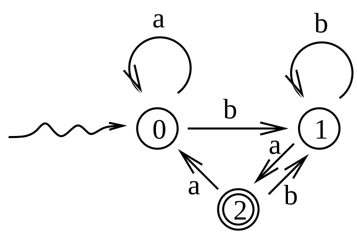
   $a^*b(aa^*b)^*$

   (b) The language accepted by the machine illustrated below.



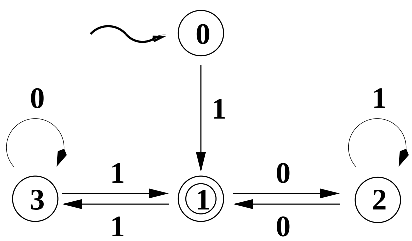
   $(1 + 01^*1)^*$

(c) The language accepted by the machine illustrated below.



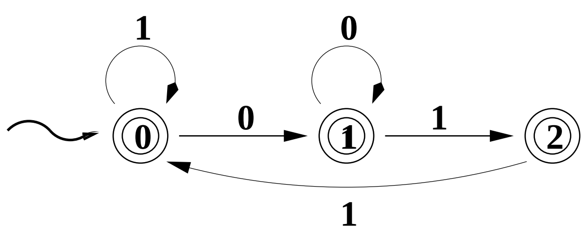
$a^*bb^*a(aa^*bbb^*a + ba)^*$

(d) The language accepted by the machine illustrated below.



$1(10^*1 + 01^*0)^*$

(e) The language accepted by the machine illustrated below. (Harder then the others.)



$(1 + 00^*11) + 1^*0(0 + 111^*0)^* + 11^*00^*1)^*$

I simply wrote regular expressions for $L(0)$, $L(1)$ and $L(2)$ and took their union ('+').

4. Recall the "pigeonhole" principle: If $f : A \to B$ is a function, where A and B are finite and $|A| > |B|$, there must exist elements $x, y \in A$, such that $x \neq y$ and $f(x) = f(y)$. Prove the pumping lemma for regular languages.

   Let L be a regular language accepted by an DFA M. Let p be the number of states of M. Let $w = w_1w_2 \dots w_\ell \in L$ where $\ell \geq p$. Let $q_0, q_1, \dots q_\ell$ be an accepting computation of M with input w, that is, the arc from q_{i-1} to q_i is labeled w_i, and q_ℓ is a final state of M. The number of states in this computation is $\ell + 1$, which is greater than the number of states of M. Thus, by the pigeonhole principle, the computation must use some state at least twice. Pick the smallest j such that $q_j = q_i$ for some $i < j$. Since that is the first collision, the j states $q_0, \dots q_{j-1}$ must be distinct, hence $j \leq p$.

   Let $x = w_1w_2 \dots w_i$, let $y = w_{i+1} \dots w_j$ and let $z = w_{j+1} \dots w_\ell$.

   We observe that $xyz = w$, $|xy| = j \leq p$, and $|y| = j - i > 0$.

   Finally the computation of M with input xy^iz is $q_0, \dots q_i(q_i + 1 \dots q_j)^i q_{j+1} \dots q_\ell$, and thus $xy^iz \in L$.

5. State the pumping lemma for context-free languages.

   For any context-free language L
   There exists an integer p, the pumping length of L, such that
   For any $w \in L$ of length at least p
   There exist strings u, v, x, y, z such that the following four statements hold:
   1. $w = uvxyz$
   2. $|vxy| \leq p$
   3. $|v| + |y| > 0$
   4. For any integer $i \geq 0$, $uv^ixy^iz \in L$.

6. Use the pumping lemma for context-free languages to prove that $L = \{a^nb^nc^n : n \geq 0\}$ is not context-free.

   Assume that L is context-free. Let p be the pumping length of L. Let $w = a^pb^pc^p \in L$. Note that $|w| = 3p \geq p$. Pick strings u, v, x, y, z such that the four concluding statements of the pumping lemma hold.

   By condition 1, $uvxyz = w = a^pb^pc^p$. By condition 3, $|v| + |y| > 0$. By condition 4, $uxz \in L$, and $|uxz| = 3p - (|v| + |y|) < 3p$.

   Let $m = |u|$. By condition 2, $|vxy| \leq p$, hence $|uvxy| \leq mp$. If vxy contains an a, then $m < p$, which implies that $|uvxy| < 2p$. In that case, vxy cannot contain any c. Similarly, if $m \geq p$, vxy cannot contain any a. Thus v and y together either contain no a or no c, which implies that either uxz contains c^p or a^p. Every member of L contains equal numbers of each of the three symbols, hence $|uxz| \geq 3p$, contradiction.

7. Give a definition of an *ambiguous* context-free grammar.

   A grammar context-free grammar G is defined to be ambiguous if there is some string in $L(G)$ which has at least two different parse trees. Equivalently, if there is some string which has two different left-most derivations or two different right-most derivations. Otherwise, the grammar is unambiguous.

8. Give a definition of an *inherently ambiguous* context-free language. Example: $\left\{a^ib^jc^k : i = j \text{ or } j = k\right\}$ is context-free and inherently ambiguous.

   A context-free language is inherently ambiguous if it is not generated by any unambiguous context-free grammar.

9. What is Russell's paradox?

   A member of a set could also be a set. Let $\mathcal{S}$ be the set of all sets which do not contain themselves as members. That is, $\mathcal{S} = \{X : X \text{ is a set and } X \notin X\}$

   We now ask whether $\mathcal{S}$ is a member of itself.

   Case 1: $\mathcal{S} \in \mathcal{S}$. Then $\mathcal{S} \notin \mathcal{S}$ by the definition of $\mathcal{S}$.

   Case 2: $\mathcal{S} \notin \mathcal{S}$. Then $\mathcal{S} \in \mathcal{S}$ by the definition of $\mathcal{S}$.

   Thus, we have a contradiction. This is called Russell's paradox, the most famous paradox in mathematics, which led to rethinking set theory. It is named after the philospher Bertrand Russell, 1870–1972.