Overview of Difficulty of Computation

The central idea of the course is the limit of computation. Most of your life you have heard that there is no such thing as an unsolvable problem: there are only problems that haven't yet been solved, and they just require more effort. I have heard from Russians who lived it that this was, in fact, Communist doctrine, and teachers had to be very careful teaching the theory of computation.[1]

Here are some examples that we will learn.

There exist problems which are unsolvable by any logical means. An example is the *halting problem.* Will a given computer program ever halt? You might think that all you have to do is run it and see. However, what if it doesn't halt within five mintues? Run it all day. What if it still doesn't halt. Run it all year. What if it still hasn't halted. Maybe it will, if you run it long enough, and maybe it won't. If not, can you prove that it will never halt? The answer is, maybe not! This fact was proved by Alan Turing in 1936.

Is $\mathcal{P}$ equal to $\mathcal{NP}$? More recently, with the introduction of complexity theory, other limits have been discovered. Given a problem P and given an instance of that problem written with n bits, can the instance be solved in $O(n)$ steps? $O(n^2)$? Maybe, maybe not. We say that a problem P is in the class $\mathcal{P}$–TIME if there is some constant k such that every instance of length (number of bits of its description) n can be solved in $O(n^k)$ steps. This is important, because, if not, large instances of P cannot be solved in a (humanly) reasonable amount of time. We will learn about a class of problems called $\mathcal{NP}$*–complete* which may or may not be $\mathcal{P}$*–time*: no one knows. These include many problems of practical importance!

Is $\mathcal{P}$ equal to $\mathcal{NC}$? With the developement of large parallel processing machines, another unsolved question has arisen. If a problem can be solved in T steps on a single processor machine, can the work be efficiently farmed out to many processors running in parallel, so that it can be worked quickly? Yes, in some cases. But in all cases? Maybe, but maybe not. Again, no one knows the answer to this very important question.

[1] Similarly, it was also Communist doctrine that crops and livestock could not be improved by selective breeding. This was disastrous for Soviet agriculture.