True/False Questions

$\mathcal{P}$ means $\mathcal{P}$–TIME

$calNcalP$ means $\mathcal{NP}$–TIME

$calRcalE$ means recursively enumerable
$\mathcal{NC}$ means Nick's class.
If $\mathcal{C}$ is any class of languages, co-$\mathcal{C}$ means the class of all languages which are complements of languages in $\mathcal{C}$.
A *binary language* is a language over the binary alphabet $\{0,1\}$.
A *recursive* function is any function which can be computed by a machine.
A *recursive* real number is any real number whose n^{th} decimal digit is a recursive function of n.
A *fraction* is a string consisting of a numeral, followed by a slash, followed by another numeral.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (i) ________ Every subset of a regular language is regular.

   (ii) ________ The union of any two regular languages must be regular

   (iii) ________ The concatenation of any two regular languages must be regular

   (iv) ________ The union of any two context-free languages must be context-free

   (v) ________ There is a polynomial time algorithm which determines whether any two regular expressions are equivalent.

   (vi) ________ If two regular expressions are equivalent, there is a polynomial time proof that they are equivalent.

   (vii) ________ The complement of any regular language is regular.

   (viii) The Kleene closure of any regular language is regular. ________

   (ix) ________ The intersection of any two regular languages is regular.

   (x) ________ Let L be the language over $\{a,b,c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (xi) ________ The language $\{a^nb^n \mid n \geq 0\}$ is context-free.

   (xii) ________ The language $\{a^nb^nc^n \mid n \geq 0\}$ is context-free.

   (xiii) ________ The language $\left\{a^ib^jc^k \mid j = i + k\right\}$ is context-free.

   (xiv) ________ The intersection of any regular language with any context-free language is context-free.

   (xv) ________ The intersection of any two context-free languages is context-free.

(xvi) ________ If L is a context-free language over an alphabet with just one symbol, then L is regular.

(xvii) ________ There is a deterministic parser for any context-free grammar.

(xviii) ________ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.

(xix) ________ Every regular language is context-free.

(xx) ________ Every context-free language is in $\mathcal{P}$.

(xxi) ________ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

(xxii) ________ The problem of whether a given string is generated by a given context-free grammar is decidable.

(xxiii) ________ If G is a context-free grammar, the question of whether $L(G) = \emptyset$ is decidable.

(xxiv) ________ The Kleene closure of any $\mathcal{NP}$ langauge is $\mathcal{NP}$

(xxv) ________ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.

(xxvi) ________ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

(xxvii) ________ The problem of whether a given context-sensitive grammar generates a given string is in the class $\mathcal{NP}$.

(xxviii) ________ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$-TIME.

(xxix) ________ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

(xxx) ________ Every undecidable problem is $\mathcal{NP}$-complete.

(xxxi) ________ Every problem that can be mathematically defined has an algorithmic solution.

(xxxii) ________ The intersection of two undecidable languages is always undecidable.

(xxxiii) ________ Every $\mathcal{NP}$ language is decidable.

(xxxiv) ________ The intersection of two $\mathcal{NP}$ languages must be $\mathcal{NP}$.

(xxxv) ________ If L_1 and L_2 are $\mathcal{NP}$-complete languages and $L_1 \cap L_2$ is not empty, then $L_1 \cap L_2$ must be $\mathcal{NP}$-complete.

(xxxvi) ________ There exists a $\mathcal{P}$-TIME algorithm which finds a maximum independent set in any graph G.

(xxxvii) ________ There exists a $\mathcal{P}$-TIME algorithm which finds a maximum independent set in any acyclic graph G.

(xxxviii) ________ $\mathcal{NC} = \mathcal{P}$.

(xxxix) ________ $\mathcal{P} = \mathcal{NP}$.

(xl) ________ $\mathcal{NP} = \mathcal{P}$-SPACE

(xli) ________ $\mathcal{P}$-SPACE = EXP-TIME

(xlii) ________ EXP-TIME = EXP-SPACE

(xliii) ________ EXP-TIME = $\mathcal{P}$-TIME.

(xliv) ________ EXP-SPACE = $\mathcal{P}$-SPACE.

(xlv) ________ $\mathcal{NP}$-SPACE = $\mathcal{P}$-SPACE.

(xlvi) ________ The traveling salesman problem (TSP) is known to be $\mathcal{NP}$-complete.

(xlvii) ________ The language consisting of all satisfiable Boolean expressions is known to be $\mathcal{NP}$-complete.

(xlviii) ________ The Boolean Circuit Problem is in $\mathcal{P}$.

(xlix) ________ The Boolean Circuit Problem is in $\mathcal{NC}$.

(l) ________ If L_1 and L_2 are undecidable langugages, there must be a recursive reduction of L_1 to L_2.

(li) ________ 2-SAT is $\mathcal{P}$-TIME.

(lii) ________ 3-SAT is $\mathcal{P}$-TIME.

(liii) ________ Primality is $\mathcal{P}$-TIME.

(liv) ________ There is a $\mathcal{P}$-TIME reduction of the halting problem to 3-SAT.

(lv) ________ Every context-free language is in $\mathcal{NC}$.

(lvi) ________ Addition of binary numerals is in $\mathcal{NC}$.

(lvii) ________ Every context-sensitive language is in $\mathcal{P}$.

(lviii) ________ Every language generated by a general grammar is recursive.

(lix) ________ The problem of whether two given context-free grammars First generate the same language is decidable.

(lx) ________

The language of all fractions (using base 10 numeration) whose values are less than π is decidable.

(lxi) ________ Any context-free language over the unary alphabet is regular.

(lxii) ________ Any context-sensitive language over the unary alphabet is regular.

(lxiii) ________ Any recursive language over the unary alphabet is regular.

(lxiv) ________ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a unary numeral.

(lxv) ________ For any two languages L_1 and L_2, if L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 must be undecidable.

(lxvi) ________ For any two languages L_1 and L_2, if L_2 is undecidable and there is a recursive reduction of L_1 to L_2, then L_1 must be undecidable.

(lxvii) ________ If P is a mathematical proposition that can be written using a string of length n, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

(lxviii) ________ If L is any $\mathcal{NP}$ language, there must be a $\mathcal{P}$–TIME reduction of L to the partition problem.

(lxix) A language is decidable if and only if it is recursive.

(lxx) ________ Every bounded function is recursive.

(lxxi) ________ If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$.

(lxxii) A language is $\mathcal{RE}$ if and only if it is generated by a grammar.

(lxxiii) ________ If L is $\mathcal{RE}$ and also co-$\mathcal{RE}$, then L must be decidable.

(lxxiv) ________ Every language is enumerable.

(lxxv) ________ If a language L is undecidable, then there can be no machine that enumerates L.

(lxxvi) ________ There exists a mathematical proposition which is true, but can be neither proved nor disproved.

(lxxvii) ________ There is a non-recursive function which grows faster than any recursive function.

(lxxviii) ________ There exists a machine that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

(lxxix) ________ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

(lxxx) ________ **Rush Hour**, the puzzle sold in game stores everywhere, generalized to a board of arbitrary size, is known to be $\mathcal{NP}$–complete.

(lxxxi) ________ Every subset of any enumerable set is enumerable.

(lxxxii) ________ The computer language Pascal has Turing power.

(lxxxiii) ________ Computing the square of an integer written in binary notation is an $\mathcal{NC}$ function.

(lxxxiv) ________ If L is any $\mathcal{P}$-TIME language, there is an $\mathcal{NC}$ reduction of L to the Boolean circuit problem.

(lxxxv) ________ If an abstract Pascal machine can perform a computation in polynomial time, there must be some Turing machine that can perform the same computation in polynomial time.

(lxxxvi) ________ The binary integer factorization problem is co-$\mathcal{NP}$.

(lxxxvii) ________ There is a polynomial time reduction of the subset sum problem to the binary numeral factorization problem.

(lxxxviii) ________ There is a polynomial time reduction of the binary numeral factorization problem to the subset sum problem.

(lxxxix) ________ For any real number x, the set of fractions whose values are less than x is $\mathcal{RE}$.

(xc) ________ For any recursive real number x, the set of fractions whose values are less than x is recursive (i.e., decidable).

(xci) ________ The concatenation of any two deterministic context-free languages must be a DCFL.

(xcii) ________ The concatenation of any two context-free languages must be context-free

(xciii) ________ The union of any two deterministic context-free languages must be a DCFL.

(xciv) ________ The intersection of any two deterministic context-free languages must be a DCFL.

(xcv) ________ The membership problem for any CFL is in the class $\mathcal{P}$-TIME.

(xcvi) ________ Every finite language is decidable.

________ Every context-free language is $\mathcal{NC}$.

(xcvii) ________ 2SAT is known to be $\mathcal{NP}$-complete.

(xcviii) ________ The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

(xcix) ________ The complement of any $\mathcal{P}$–SPACE language is $\mathcal{P}$–SPACE.

(c) ________ The complement of any decidable language is decidable.

(ci) The complement of any undecidable language is undecidable. ________

(cii) The complement of any $\mathcal{RE}$ language is $\mathcal{RE}$. ________

The *jigsaw puzzle problem* is, given a set of various polygons, and given a rectangular table, is it possibe to assemble those polygons to exactly cover the table?

The *furniture mover's problem* is, given a room with a door, and given a set of objects outside the room, it is possible to move all the objects into the room through the door?

(ciii) ________ The jigsaw puzzle problem is known to be $\mathcal{NP}$ complete.

(civ) ________ The jigsaw puzzle problem is known to be $\mathcal{P}$–SPACE complete.

(cv) ________ The furniture mover's problem is known to be $\mathcal{NP}$ complete.

(cvi) ________ The furniture mover's problem is known to be $\mathcal{P}$–SPACE complete.

(cvii) ________ The complement of any recursive language is recursive.

(cviii) ________ The complement of any undecidable language is undecidable.

(cix) ________ Every undecidable language is either $\mathcal{RE}$ or co-$\mathcal{RE}$.

(cx) ________ For any infinite countable sets A and B, there is a 1-1 correspondence between A and B.

(cxi) ________ The set of all binary languages is countable.

(cxii) ________ A language L is recursively enumerable if and only if there is a machine which accepts L.

(cxiii) ________ A language is $\mathcal{RE}$ if and only if it is accepted by some machine.

(cxiv) ________ Every $\mathcal{NP}$ language is reducible to the independent set problem in polynomial time.

(cxv) ________ If a Boolean expression is satisfiable, there is a polynomial time proof that it is satisfiable.

(cxvi) ________ The general sliding block problem is $\mathcal{P}$–SPACE complete.

(cxvii) ________The regular expression equivalence problem is $\mathcal{P}$–SPACE complete.

(cxviii) ________The context-sensitive membership problem is $\mathcal{P}$–SPACE complete.

(cxix) ________The Post correspondence problem is decidable.

(cxx) ________The halting problem is decidable.