University of Nevada, Las Vegas Computer Science 456/656 Fall 2024

Final Examination December 11, 2024

Name:_____

The entire test is 525 points.

No books, notes, machines, or scratch paper. Use pen or pencil, any color. Use the backs of the pages for scratch paper. If you need more scratch paper, it will be provided. If you want a piece of scratch paper graded, write "See scratch paper," on the test, and "Grade this page," and your name, on the scratch paper.

1. True/False. T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (i) [5 points] _____ The set of palindromes over $\{a, b\}$ is accepted by some DPDA.

   (ii) [5 points] _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

   (iii) [5 points] _____ Every problem that can be mathematically defined has an algorithmic solution.

   (iv) [5 points] _____ $\mathcal{NC} = \mathcal{P}$.

   (v) [5 points] _____ $\mathcal{P} = \mathcal{NP}$.

   (vi) [5 points] _____ co-$\mathcal{NP} = \mathcal{NP}$.

   (vii) [5 points] _____ A language is recursively enumerable if and only if it is generated by an unrestricted (general) grammar.

   (viii) [5 points] _____ If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$–TIME.

   (ix) [5 points] _____ If L is in $\mathcal{RE}$ and also co-$\mathcal{RE}$, then L must be recursive (decidable).

   (x) [5 points] _____ There is a function which grows faster than any computable function.

   (xi) [5 points] _____ There is a machine which writes the decimal expansion of π.

   (xii) [5 points] _____ The binary integer factorization problem is co-$\mathcal{NP}$.

   (xiii) [5 points] _____ If L is $\mathcal{NP}$, there is a polynomial time reduction of L to the subset sum problem.

   (xiv) [5 points] _____ The intersection of any two $\mathcal{NP}$ languages is $\mathcal{NP}$.

   (xv) [5 points] _____ Multiplication of matrices with binary numeral entries is $\mathcal{NC}$.

   (xvi) [5 points] _____ Equivalence of context-free grammars is $\mathcal{RE}$.

   (xvii) [5 points] _____ The language of all true mathematical statements is recursively enumerable.

   (xviii) [5 points] _____ If there exists a polynomial time algorithm for any $\mathcal{NP}$–complete problem, then $\mathcal{P} = \mathcal{NP}$.

(xix) [5 points] _____ RSA encryption is accepted as secure by experts, because they believe that the factorization problem is very hard.

(xx) [5 points] _____ For any real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

(xxi) [5 points] _____ If a Boolean expression is satisfiable, there is a $\mathcal{P}$–TIME proof that it's satisfiable.

(xxii) [5 points] _____ If the Boolean circut problem (CVP) is $\mathcal{NC}$, then $\mathcal{P} = \mathcal{NC}$.

(xxiii) [5 points] _____ If L is any $\mathcal{P}$–TIME language, there is an $\mathcal{NC}$ reduction of L to CVP, the Boolean circuit problem.

(xxiv) [5 points] _____ If L is a $\mathcal{P}$–TIME language, there is a Turing Machine which decides L in polynomial time.

(xxv) [5 points] _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is $\mathcal{NC}$.

(xxvi) [5 points] _____ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.

(xxvii) [5 points] _____ The problem of whether a given string is generated by a given context-free grammar is $\mathcal{NC}$.

(xxviii) [5 points] _____ There is an unambiguous context-free grammar for the language of palindromes over $\{a, b\}$.

(xxix) [5 points] _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

(xxx) [5 points] _____ EXP-TIME = $\mathcal{P}$-TIME.

(xxxi) [5 points] _____ EXP-SPACE = $\mathcal{P}$-SPACE.

(xxxii) [5 points] _____ Every context-free language is in $\mathcal{NC}$.

(xxxiii) [5 points] _____ The language of all fractions (using binary numeration) whose values are less than π is decidable.

(xxxiv) [5 points] _____ For any two languages L_1 and L_2, if L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 must be undecidable.

(xxxv) [5 points] _____ Every undecidable language is either $\mathcal{RE}$ or co-$\mathcal{RE}$.

(xxxvi) [5 points] _____ There is a polynomial time algorithm which determines whether any two regular expressions are equivalent.

(xxxvii) [5 points] _____ 2-SAT is $\mathcal{NP}$-complete.

(xxxviii) [5 points] _____ The computer language C++ has Turing power.

2. [10 points] State the pumping lemma for regular languages.

3. [20 points] Prove that any language accepted by any machine can be enumerated by some other machine.

4. Every language, or problem, falls into exactly one of these categories. For each of the languages, write a letter indicating the correct category.

   A Known to be $\mathcal{NC}$.
   B Known to be $\mathcal{P}$–TIME, but not known to be $\mathcal{NC}$.
   C Known to be $\mathcal{NP}$, but not known to be $\mathcal{P}$–TIME and not known to be $\mathcal{NP}$–complete.
   D Known to be $\mathcal{NP}$–complete.
   E Known to be $\mathcal{P}$–SPACE but not known to be $\mathcal{NP}$
   F Known to be EXP–TIME but not nown to be $\mathcal{P}$–SPACE.
   G Known to be EXP–SPACE but not nown to be EXP–TIME.
   H Known to be decidable, but not nown to be EXP–SPACE.
   K $\mathcal{RE}$ but not decidable.
   L co-$\mathcal{RE}$ but not decidable.
   M Neither $\mathcal{RE}$ nor co-$\mathcal{RE}$.

   (i) [5 points] _____ The set of binary numerals for prime numbers.

   (ii) [5 points] _____ 3-SAT.

   (iii) [5 points] _____ The Boolean Circuit Probem, also known as the Circuit Value Problem.

   (iv) [5 points] _____ The Independent Set problem.

   (v) [5 points] _____ The Subset Sum Problem.

(vi) [5 points] _____ All positions of RUSH HOUR from which it is possible to win.

(vii) [5 points] _____ All C++ programs which halt if given themselves as input.

(viii) [5 points] _____ The set of all equivalent pairs of context-free grammars.

(ix) [5 points] _____ The traveling salsman problem (TSP).

(x) [5 points] _____ The Jigsaw problem. (That is, given a finite set of two-dimensional pieces, can they be assembled into a rectangle, with no overlap and no spaces.)

(xi) [5 points] _____ Factorization of binary numerals.

(xii) [5 points] _____ All pairs $(\langle M \rangle, w)$ such that M is a machine which halts with input w.

(xiii) [5 points] _____ All satisfiable Boolean expressions.

(xiv) [5 points] _____ All pairs of binary numerals $(\langle p \rangle, \langle q \rangle)$ such that p has a divisor greater than q.

5. Give a definition of each term.

(a) [10 points] Accept. (That is, what does it mean for a machine to accept a language.)

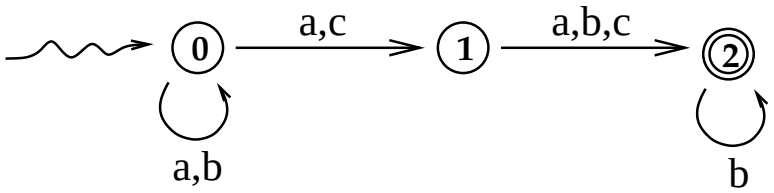
(b) [10 points] Decide. (That is, what does it mean for a machine to decide a language.)

(c) [10 points] Canonical order of a language L.

(d) [10 points] Recursive real number.

(e) [10 points] Nick's Class.

6. [10 points] Give a regular expression for the language accepted by the machine in the figure below.



7. Which class of languages does each of these machine classes accept?

   (a) [5 points] Deterministic finite automata. _____

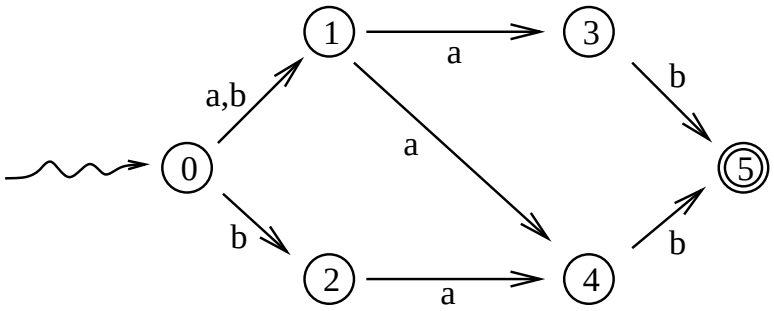
   (b) [5 points] Non-deterministic finite automata. _____

   (c) [5 points] Push-down automata. _____

   (d) [5 points] Turing Machines. _____

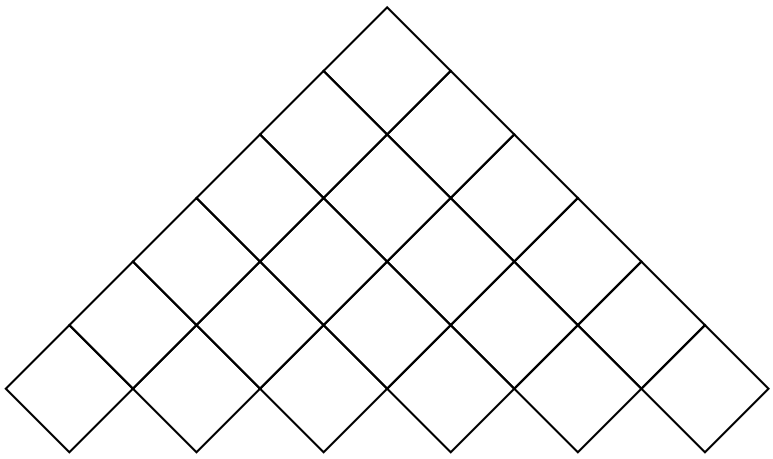
8. [10 points] Let L be the binary language which consists of all strings which contain the substring 101. Construct a DFA which accepts L.

9. [20 points] Construct a minimal DFA equivalent to the NFA shown below.



10. [20 points] Use the CYK algorithm to decide whether $x - x - -x$ is generated by the CNF grammar below, by filling in the matrix.

$E \to ME$
$A \to EM$
$E \to AE$
$M \to -$
$E \to x$



11. [20 points] Give a polynomial time reduction of the subset sum problem to partition.

12. [20 points] Prove that the halting problem is undecidable.

13. Label each of the following sets as countable or uncountable.

    (a) [5 points] _____ The set of integers.

    (b) [5 points] _____ The set of rational numbers.

    (c) [5 points] _____ The set of real numbers.

    (d) [5 points] _____ The set of binary languages.

    (e) [5 points] _____ The set of co-$\mathcal{RE}$ binary languages.

    (f) [5 points] _____ The set of sets of integers.

    (g) [5 points] _____ The set of recursive real numbers.

14. [10 points] What is the Church-Turing Thesis?

15. [20 points] The grammar below is an ambiguous CF grammar and is parsed by the LALR parser whose Action and Goto tables are shown. Write a computation of the parser for the input string *iiwaea*.

1. $S \to i_2 S_3$
2. $S \to i_2 S_3 e_4 S_5$
3. $S \to w_6 S_7$
4. $S \to a_8$

	ACTION					GOTO
	a	i	e	w	\$	S
0	$s8$	$s2$		$s6$		1
1					halt	
2	$s8$	$s2$		$s6$		3
3			$s4$		$r1$	
4	$s8$	$s2$		$s6$		5
5			$r2$		$r2$	
6	$s8$	$s2$		$s6$		7
7			$r3$		$r3$	
8			$r4$		$r4$	