University of Nevada, Las Vegas Computer Science 456/656 Spring 2024

Final Examination May 8, 2024

The entire test is 500 points.

No books, notes, or electronic devices, or scratch paper. Scratch paper will be provided. Write, "Grade this page," and your name, on any scratch paper you want graded and staple it to the test. Also write, "See scratch paper," on the test at the appropriate place.

1. True/False/Open

   (i) [5 points] ________ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

   (ii) [5 points] ________ The Boolean Circuit Problem is $\mathcal{NC}$.

   (iii) [5 points] ________ Addition of binary numerals is $\mathcal{NC}$.

   (iv) [5 points] ________ The language of all fractions (using base 10 numeration) whose values are less than π is decidable.

   (v) [5 points] ________ If L is $\mathcal{RE}$ and co-$\mathcal{RE}$, then L is decidable.

   (vi) [5 points] ________ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

   (vii) [5 points] ________ There exists a mathematical proposition that can be neither proved nor disproved.

   (viii) [5 points] ________ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{NC}$.

   (ix) [5 points] ________ The complement of any undecidable language is undecidable.

   (x) [5 points] ________ co-$\mathcal{P} = \mathcal{P}$.

   (xi) [5 points] ________ The set of binary numerals for prime numbers is $\mathcal{P}$-TIME.

   (xii) [5 points] ________ There is a mathematical proposition that is true but cannot be proved true.

   (xiii) [5 points] ________ If L is $\mathcal{NP}$, there is a polynomial time reduction of L to SAT.

   (xiv) [5 points] ________ If two CF grammars are not equivalent, there is a proof that they are not equivalent.

   (xv) [5 points] ________ The language of all true mathematical statements is recursively enumerable.

   (xvi) [5 points] ________ If there exists a polynomial time algorithm for any $\mathcal{NP}$–complete problem, then $\mathcal{P} = \mathcal{NP}$.

   (xvii) [5 points] ________ A real number x is recursive if and only if the set of fractions whose values are greater than x is recursive (decidable).

(xviii) [5 points] ________ If the Boolean circuit problem (CVP) is $\mathcal{NC}$, then $\mathcal{P} = \mathcal{NC}$.

(xix) [5 points] ________ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.

(xx) [5 points] ________ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.

(xxi) [5 points] ________ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$-TIME.

(xxii) [5 points] ________ $\mathcal{NP} = \mathcal{P}$-SPACE

(xxiii) [5 points] ________ EXP-SPACE = $\mathcal{P}$-SPACE.

(xxiv) [5 points] ________ The traveling salesman problem (TSP) is $\mathcal{NP}$-complete.

(xxv) [5 points] ________ The knapsack problem is $\mathcal{P}$-TIME.

(xxvi) [5 points] ________ Every context-free language is in $\mathcal{NC}$.

(xxvii) [5 points] ________ For any two languages L_1 and L_2, if L_2 is undecidable and there is a recursive reduction of L_1 to L_2, then L_1 must be undecidable.

(xxviii) [5 points] ________ If L is any $\mathcal{NP}$ language, there must be a $\mathcal{P}$–TIME reduction of L to Boolean satisfiability.

(xxix) [5 points] ________ The intersection of any two $\mathcal{NP}$ languages is $\mathcal{NP}$.

(xxx) [5 points] ________ The independent set problem is $\mathcal{P}$-TIME.

(xxxi) [5 points] ________ Multiplication of matrices with binary numeral entries is $\mathcal{NC}$.

(xxxii) [5 points] ________ If L is any $\mathcal{P}$–TIME language, there is an $\mathcal{NC}$ reduction of L to CVP, the Boolean circuit problem.

(xxxiii) [5 points] ________ RSA encryption is believed to be secure because it is believed that the factorization problem for integers is very hard.

2. Every language, or problem, falls into exactly one of these categories. For each of the languages, write a letter indicating the correct category.

   A Known to be $\mathcal{NC}$.
   B Known to be $\mathcal{P}$–TIME, but not known to be $\mathcal{NC}$.
   C Known to be $\mathcal{NP}$, but not known to be $\mathcal{P}$–TIME and not known to be $\mathcal{NP}$–complete.
   D Known to be $\mathcal{NP}$–complete.
   E Known to be $\mathcal{P}$–SPACE but not known to be $\mathcal{NP}$
   F Known to be EXP–TIME but not nown to be $\mathcal{P}$–SPACE.
   G Known to be EXP–SPACE but not nown to be EXP–TIME.
   H Known to be decidable, but not nown to be EXP–SPACE.
   I $\mathcal{RE}$ but not decidable.
   J co-$\mathcal{RE}$ but not decidable.
   K Neither $\mathcal{RE}$ nor co-$\mathcal{RE}$.

   (i) [5 points] ________ The tiling problem. (That is, given a finite set of two-dimensional pieces, can they be placed on a rectangle with no overlap?)

   (ii) [5 points] ________ Factorization of binary numerals.

   (iii) [5 points] ________ 2-SAT.

   (iv) [5 points] ________ All configurations of RUSH HOUR from which it's possible to win.

   (v) [5 points] ________ The context-free grammar equivalence problem.

3. [10 points] Prove that every decidable language is enumerated in canonical order by some machine.

4. [10 points] Prove that every language that is enumerated in canonical order by some machine is decided by some other machine.

5. [10 points] Prove that any language which is enumerated by some machine is accepted by some other machine.

6. [10 points] I have repeatedly stated in class that no language that has parentheses can be regular. For that to be true, there must be parenthetical strings of arbitrary nesting depth. (If you don't know what nesting depth is, look it up.)

   Some programming languages have limitations on nesting depth. For example, I have read that ABAP has maximum nesting depth of 256. (Who would ever want to go that far!)

   The Dyck language is generated by the following context-free grammar. (As usual, to make grading easier, I use a and b for left and right parentheses.)

   1. $S \to aSbS$
   2. $S \to \lambda$

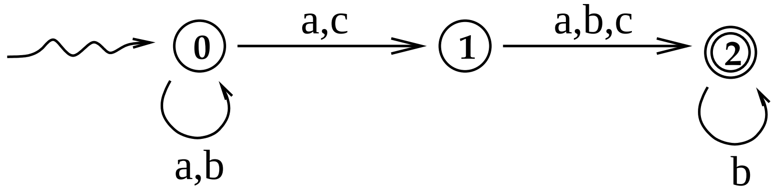
7. [10 points] Let D be any positive integer. Let L be the language consisting of all members of the Dyck language whose nesting depth does not exceed D. Prove that L is regular.

8. Give a definition of each term.

   (i) [10 points] Accept. (That is, what does it mean for a machine M to accept a language L over an alphabet Σ.)

(ii) [10 points] Decide. (That is, what does it mean for a machine M to decide a language L over an alphabet Σ.)

9. [5 points] Give a regular grammar with no more than three variables for the language accepted by the machine shown below.



NFA for Problem 9

10. Which class of languages does each of these machine classes accept?

(i) [5 points] ______________ Deterministic finite automata.

(ii) [5 points] ______________ Non-deterministic finite automata.

(iii) [5 points] ______________ Push-down automata.

(iv) [5 points] ______________ Turing Machines.

11. [10 points] Give a regular expression for the language accepted by the machine in Figure 1

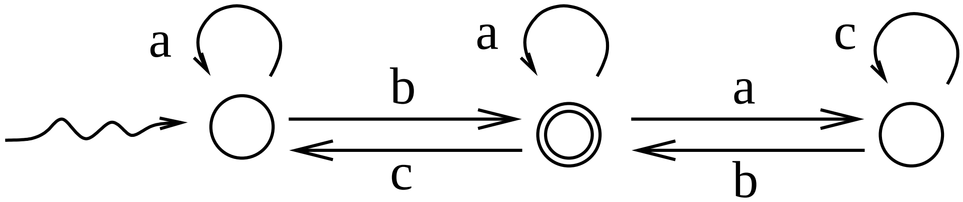


Figure 1: NFA for problem 11.

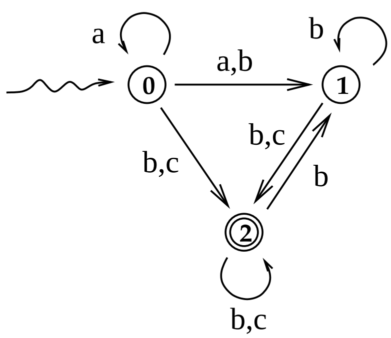
12. [10 points] Let $L = \{w \in \{a,b\}^* : \#_a(w) = \#_b(w)\}$, which is generated by the following context-free grammar.

1. $S \to aSbS$
2. $S \to bSaS$
3. $S \to \lambda$

Draw a PDA which accepts L.

13. [10 points] Find an NFA with at most 4 states which accepts the language of binary strings which contain the substring 111.

14. [20 points] Construct a minimal DFA equivalent to the NFA shown below.

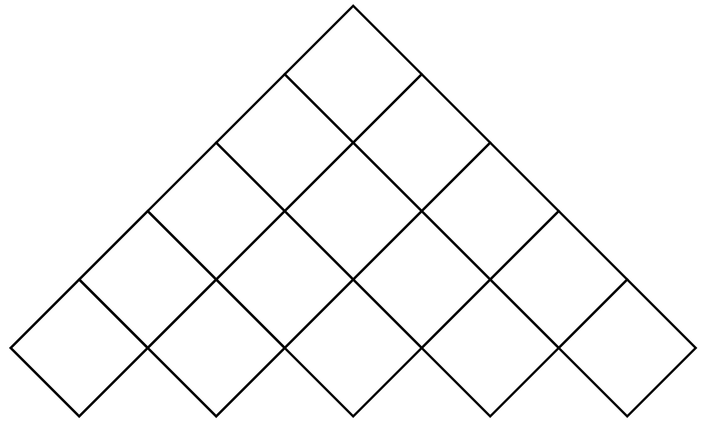


15. [20 points] Find an NFA which accepts the language generated by this grammar.

$S \to aA|cS|cC$
$A \to aA|bS|cB|\lambda$
$B \to aA|cB|bC|\lambda$
$C \to aB$

16. [20 points] Use the CYK algorithm to decide whether $abcab$ is generated by the CNF grammar:

$S \to AB \mid BC \mid CA$
$A \to a$
$B \to SA \mid SS \mid b$
$C \to c$
by filling in the matrix.



17. [10 points] State the pumping lemma for regular languages.

18. [10 points] Give the verifier-certificate definition of the class $\mathcal{NP}$.

19. [10 points] What is the importance nowadays of $\mathcal{NC}$?

20. [20 points] Give a polynomial time reduction of the subset sum problem to the partition problem.

21. Label each of the following sets as countable or uncountable.

(i) [5 points] The set of integers.

(ii) [5 points] The set of rational numbers.

(iii) [5 points] The set of real numbers.

(iv) [5 points] The set of co-$\mathcal{RE}$ binary languages.

(v) [5 points] The set of recursive real numbers.

22. [10 points] Give a context-sensitive language which is not context-free.

23. [20 points] Consider the CF grammar below. The ACTION and GOTO tables of an LALR parser for this grammar are given below, except that six actions are mising, indicated by question marks. Fill in the missing actions (below the question marks). The actions of your table must be consistent with the precedence of operators in C++.

1. $E \to E -_2 E_3$
2. $E \to -_4 E_5$
3. $E \to E *_6 E_7$
4. $E \to x_8$

	x	$-$	$*$	\$	E
0	$s8$	$s4$			1
1		$s2$	$s6$	HALT	
2	$s8$	$s4$			3
3		?	?	$r1$	
4	$s8$	$s4$			5
5		?	?	$r2$	
6	$s8$	$s4$			7
7		?	?	$r3$	
8	$s8$	$r4$	$r4$	$r4$	

24. [20 points] Prove that the halting problem is undecidable.