University of Nevada, Las Vegas Computer Science 456/656 Spring 2025

Final Examination May 14, 2025

Name:____

No books, notes, scratch paper, or devices. Use pen or pencil, any color. Use the backs of the pages for scratch paper. If you need more scratch paper, it will be provided. If you want a piece of scratch paper to be graded, write, "Grade this page." and staple it to your test. Also write, "See scratch paper" on the test.

The entire test is 525 points.

Throughout, $\mathcal{P}$ means $\mathcal{P}$–TIME.

1. {5} True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (i) {5} ______ The complement of any context-free language is context-free.

   (ii) {5} ______ The complement of any $\mathcal{P}$ language is $\mathcal{P}$.

   (iii) {5} ______ The complement of any decidable language is decidable.

   (iv) {5} ______ The complement of any $\mathcal{RE}$ language is $\mathcal{RE}$.

   (v) {5} ______ The intersection of two undecidable languages is undecidable.

   (vi) {5} ______ The intersection of two $\mathcal{NC}$ languages is $\mathcal{NC}$.

   (vii) {5} ______ $\mathcal{NC} = \mathcal{P}$.

   (viii) {5} ______ $\mathcal{P} = \mathcal{NP}$.

   (ix) {5} ______ If every $w \in L$ can be proved to be in L, then L must be decidable.

   (x) {5} ______ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

   (xi) {5} ______ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (xii) {5} ______ Every EXP–SPACE-complete language is decidable.

   (xiii) {5} ______ The language consisting of all satisfiable Boolean expressions is $\mathcal{NP}$-complete.

   (xiv) {5} ______ The Boolean Circuit Problem is in $\mathcal{NC}$.

   (xv) {5} ______ 2-SAT is $\mathcal{P}$–TIME

   (xvi) {5} ______ 3-SAT is $\mathcal{P}$-TIME.

   (xvii) {5} ______ Primality, using binary numerals, is $\mathcal{P}$-TIME.

   (xviii) {5} ______ Every context-free language is in $\mathcal{NC}$.

(xix) {5} ______ The problem of whether two given context-free grammars generate the same language is decidable.

(xx) {5} ______ The language of all fractions (using base 10 numeration) whose values are less than π is decidable.

(xxi) {5} ______ For any two languages L_1 and L_2, if L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 must be undecidable.

(xxii) {5} ______ If L is any $\mathcal{NP}$ language, there must be a $\mathcal{P}$–TIME reduction of L to the Partition problem.

(xxiii) {5} ______ If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$.

(xxiv) {5} ______ If a language L is in $\mathcal{RE}$ and also in co-$\mathcal{RE}$, then L must be decidable.

(xxv) {5} ______ There is a polynomial time algorithm which determines whether any two regular expressions are equivalent.

(xxvi) {5} ______ The independent set problem is $\mathcal{P}$-TIME.

(xxvii) {5} ______ If anyone ever finds a polynomial time algorithm for any $\mathcal{NP}$–complete language, then we will know that $\mathcal{P} = \mathcal{NP}$.

(xxviii) {5} ______ RSA encryption is believed to be secure because it is believed that the factorization problem for integers is very hard.

(xxix) {5} ______ If L is $\mathcal{RE}$ and $w \in L$, there is a proof that $w \in L$.

(xxx) {5} ______ There is a mathematical proposition that is true but cannot be proved true.

(xxxi) {5} ______ If a Boolean expression is satisfiable, there is a $\mathcal{P}$–TIME proof that it's satisfiable.

(xxxii) {5} ______ If the Boolean circuit problem (CVP) is $\mathcal{NC}$, then $\mathcal{P} = \mathcal{NC}$.

2. {10} Give a definition of *Accept.* (That is, what does it mean for a machine to accept a language.)

3. {10} State the pumping lemma for context-free languages.

4. {10} State the Church-Turing thesis.

5. {10} What is the class of languages accepted by push-down automata?

6. Every language, or problem, falls into exactly one of these categories. For each of the languages, write a letter indicating the correct category.

   A Known to be $\mathcal{NC}$.
   B Known to be $\mathcal{P}$–TIME, but not known to be $\mathcal{NC}$.
   C Known to be $\mathcal{NP}$, but not known to be $\mathcal{P}$–TIME and not known to be $\mathcal{NP}$–complete.
   D Known to be $\mathcal{NP}$–complete.
   E Known to be $\mathcal{P}$–SPACE but not known to be $\mathcal{NP}$
   F Known to be EXP–TIME but not known to be $\mathcal{P}$–SPACE.
   G Known to be EXP–SPACE but not known to be EXP–TIME.
   H Known to be decidable, but not known to be EXP–SPACE.
   I $\mathcal{RE}$ but not decidable.
   K co-$\mathcal{RE}$ but not decidable.
   L Neither $\mathcal{RE}$ nor co-$\mathcal{RE}$.

   (i) {5} ________ All binary numerals for composite integers. (Composite means not prime.)
   (ii) {5} ________ Boolean matrix multiplication.
   (iii) {5} ________ All C++ programs which do not halt if given themselves as input.
   (iv) {5} ________ 2-SAT.
   (v) {5} ________ The Independent Set problem.
   (vi) {5} ________ The Sliding block problem.
   (vii) {5} ________ The Traveling salesman problem.
   (viii) {5} ________ The Graph isomorphism problem.

(ix) {5} ________ The 3-coloring problem.

(x) {5} ________ The 2-coloring problem.

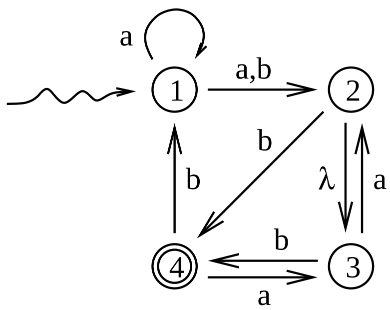
(xi) {5} ________ The set of binary numerals for Busy Beaver numbers.

7. {20} Prove that any language which is enumerated by some machine is accepted by some other machine.

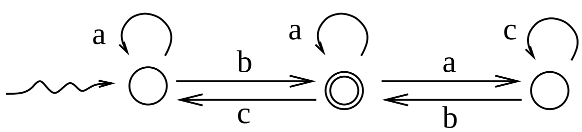
8. {10} Find a DFA with 4 states which accepts the language of all binary strings which contain the substring 101.

9. {20} Give a polynomial time reduction of the subset sum problem to the partition problem.

10. {20} Construct a minimal DFA equivalent to the NFA shown below.



11. {20} Give a regular expression which describes the language accepted by the NFA shown in the figure below.



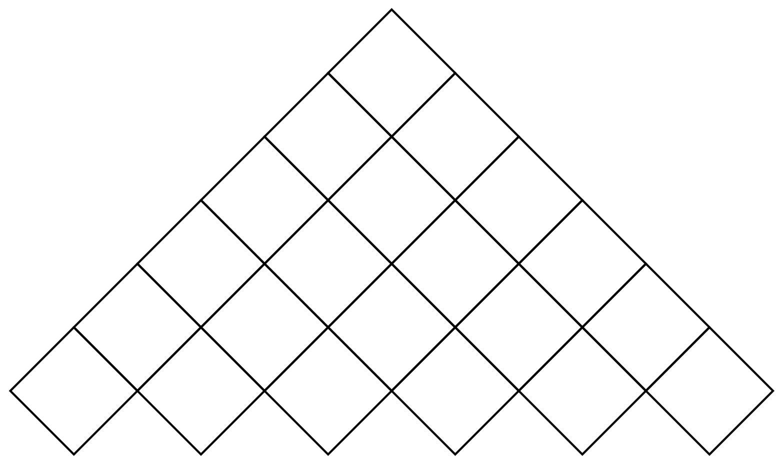
12. {10} Let $L = \{w \in \{a,b\}^* : \#_a(w) = \#_b(w)\}$, Draw a DPDA which accepts L. (Recall that the input to that DPDA must be of the form $w\$$, where $w \in L$ and \$ is the end-of-file symbol.)

13. {10} We know that context-free languages are exactly those which are accepted by push-down automata. We now define a new class of machines, which we call "limited push-down automata." An LPDA is exactly the same as a PDA, but with the restriction that the stack is never allowed to be larger than some given constant. What is the class of languages accepted by limited push-down automata? Think!

14. {10} What is the class of languages decided by 2-PDA? A 2-PDA is the same as a PDA, except that it has two stacks instead of just one.

15. {10} Use the CYK algorithm to decide whether $x - x - - x$ is generated by the CNF grammar below, by filling in the matrix. The start symbol is E.

$E \to ME$
$A \to EM$
$E \to AE$
$M \to -$
$E \to x$



16. Label each of the following sets as countable or uncountable. Write **C** or **U**.

(i) {5} ________ The set of integers.
(ii) {5} ________ The set of rational numbers.
(iii) {5} ________ The set of real numbers.
(iv) {5} ________ The set of binary languages.
(v) {5} ________ The set of co-$\mathcal{RE}$ binary languages.
(vi) {5} ________ The set of undecidable binary languages.
(vii) {5} ________ The set of functions from integers to integers.
(viii) {5} ________ The set of recursive real numbers.
(ix) {5} ________ The set of $\mathcal{P}$–SPACE languages over the binary alphabet.
(x) {5} ________ The set of functions from the integers to the binary alphabet $\{0, 1\}$.
(xi) {5} ________ The set of functions from the binary alphabet $\{0, 1\}$ to the integers.

17. {20} The grammar below is an ambiguous CF grammar with start symbol E, and is parsed by the LALR parser whose ACTION and GOTO tables are shown here. The ACTION table is missing actions for the second column, when the next input symbol is the "minus" sign. Fill it in. Remember the C++ precedence of operators. (Hint: the column has seven different actions: s2, s4, r1, r2, r3, r4, and r5, some more than once, and has no blank spaces.)

1. $E \to E -_2 E_3$
2. $E \to -_4 E_5$
3. $E \to E *_6 E_7$
4. $E \to (_8 E_9)_{10}$
5. $E \to x_{11}$

	x	$-$	$*$	(	)	\$	S
0	$s11$			$s8$			1
1			$s6$			halt	
2	$s11$			$s8$			3
3			$s6$		$r1$	$r1$	
4	$s11$			$s8$			5
5			$r2$		$r2$	$r2$	
6	$s11$			$s8$			7
7			$r3$		$r3$	$r3$	
8	$s11$			$s8$			9
9			$s6$		$s10$		
10			$r4$		$r4$	$r4$	
11			$r5$		$r5$	$r5$	

18. (i) {10} Give a context-sensitive grammar for $\{a^n b^n c^n : n > 0\}$

(ii) {10} Using that grammar, give a derivation of the string *aaabbbccc*.

19. {20} The grammar given below generates the Dyck language, and an LALR parser for that grammer is given to the right:

1. $S \to a_2 S_3 b_4 S_5$
2. $S \to \lambda$

	ACTION			GOTO
	a	b	$	S
0	s2			1
1			HALT	
2	s2	r2		3
3		s4		
4	s2	r2	r2	5
5		r1	r1	

Walk through the actions of the LALR parser for the input string *abab*.

stack	input	output	action
$\$_0$	$abab\$$		

20. {20} Prove that the Halting problem is undecidable.