CSC 456/656 Fall 2025 Answers toFirst Examination September 18, 2025

The entire exam is 235 points.

Name:____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

In the questions of this test, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.
If L is a language over an alphabet Σ, we define the *complement* of L to be the set of all strings over Σ which are not in L. If $\mathcal{C}$ is a class of languages, we define co-$\mathcal{C}$ to be the class of all complements of members of $\mathcal{C}$.

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

(i) **F** Every subset of a regular language is regular.

(ii) **T** The complement of a regular language is always a regular language.

(iii) **T** The class of regular languages is closed under union.

(iv) **T** The Kleene closure of any CFL is a CFL.

(v) **T** The class of regular languages is closed under intersection.

(vi) **T** The class of context-free languages is closed under union.

(vii) **F** The class of context-free languages is closed under intersection.

(viii) **F** $\{a^n b^n : n \geq 0\}$ is regular.

(ix) **O** $\mathcal{P} = \mathcal{NP}$.

(x) **T** The complement of any $\mathcal{P}$–SPACE language is $\mathcal{P}$–SPACE.

(xi) **O** If L is both $\mathcal{NP}$ and co–$\mathcal{NP}$, then L must be $\mathcal{P}$–TIME.

(xii) **F** Given a regular language L, there is a unique minimal NFA which accepts L.

(xiii) **T** If L_1 is a $\mathcal{NP}$-complete language and there is a $\mathcal{P}$–TIME reduction of L_1 to L_2, then L_2 must be $\mathcal{NP}$-hard.

(xiv) **F** If L_2 is a $\mathcal{NP}$-complete language and there is a $\mathcal{P}$–TIME reduction of L_1 to L_2, then L_1 must be $\mathcal{NP}$-hard.

(xv) **T** The Kleene closure of any regular language is regular.

(xvi) **T** co-$\mathcal{P} = \mathcal{P}$.

(xvii) **O** co-$\mathcal{NP} = \mathcal{NP}$.

(xviii) **T** Block placement problems, such as the jigsaw puzzle problem, are $\mathcal{NP}$.

(xix) **T** Sliding block problems, such as **Rush Hour** and the furniture mover's problem, are $\mathcal{P}$–SPACE.

(xx) **O** $\mathcal{P}$–SPACE $= \mathcal{NP}$

(xxi) **T** Every regular language is context-free.

(xxii) **F** The programming language C++ is context-free.

(xxiii) **F** The complement of any context-free language is context-free.

(xxiv) **T** Given any regular grammar G with n variables, there exists an NFA with n states that accepts $L(G)$.

(xxv) **O** Given an integer n, it is possible to find the prime factors of in $O(\log^k n)$ time for some constant k.

(xxvi) **T** Given an integer n, it is possible to decide whether n is prime in $O(\log^k n)$ time for some constant k.

(xxvii) **F** Every context-free language is accepted by some DPDA.

(xxviii) **T** Any language consisting of all decimal numerals of an arithmetic sequence (for example: $L = \{\langle 5+8n\rangle : n \geq 0\} = \{5, 13, 21, 29, 37, 45 \ldots\}$) is regular. Note: the members of L are numerals, not numbers.

(xxix) **T** Let L_1 be a regular binary language. Let L_2 be the language of all strings obtained from members of L_1 by substituting ab for 0 and c for 1. Then L_2 must be regular. For example, if $L_1 = \{0, 10, 10011\}$ then $L_2 = \{ab, cab, cababcc\}$.

2. [10 points] Give two context-free languages whose intersection is not context-free.

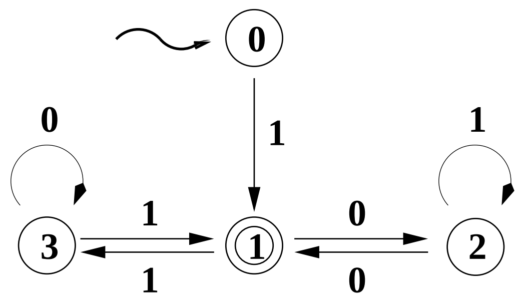
Let $L_1 = \left\{a^n b^n c^k\right\}$ and $L_2 = \left\{a^n b^k c^k\right\}$. Both are context free, but $L_1 \cap L_2 = \{a^n b^n c^n\}$ which is not. Most students used m and n or i and j, and many wrote those letters so that I needed a magnifying glass to check the answers!

3. [10 points] Give a context-free grammar for the language $\{a^n b^m c^n : n \geq 0, m \geq 0\}$

$S \to aSc$
$S \to B$
$B \to bB$
$B \to \lambda$

The most common mistake was to have only one variable S, and to use S to generate the b's. That is incorrect.

4. [5 points] Let L be the set of binary numerals for positive integers which are equivalent to 1 modulo 3. (n such that $n\%3 = 1$) That is, $L = \{1, 100, 111, 1010, 1101, \ldots\}$. Draw a DFA that accepts L.

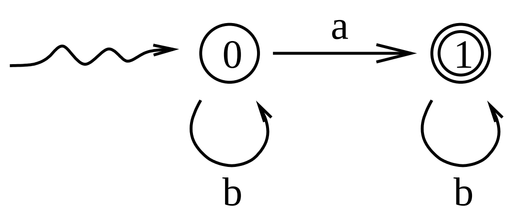


You want to avoid leading zeros.

5. [5 points] What is $\emptyset^*$, the Kleene closure of the empty language?

$\{\lambda\}$.

6. [5 points] Draw a DFA which accepts $\{b^i ab^j : i, j \geq 0\}$, the language of all strings over $\{a, b\}$ which contain exactly one a.



7. For each regular expression below, draw a DFA which accepts the language desribed by that regular expression. (5 points each)

The most common mistake was to use λ-transitions. I specifically asked for determinisitic automata.

(i) λ

(ii) a^*

(iii) $\emptyset$

(iv) a^*ba^*

(v) $(ab)^*$

(vi) a^*b^*

(vii) $(a+b)^*$

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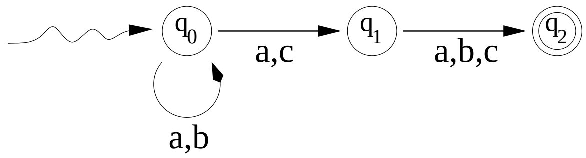
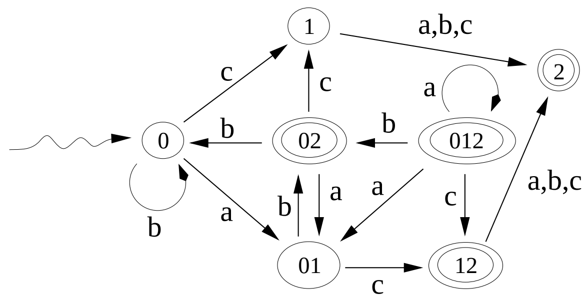
8. [10 points] Draw a minimal DFA equivalent to the following three state NFA.



	a	b	c
0	01	0	1
1	2	2	2
2	$\emptyset$	$\emptyset$	$\emptyset$
01	012	02	12
02	01	0	1
12	2	2	2
012	012	02	12



There are actually $2^3 = 8$ states in the DFA. The dead state, the empty set, is not shown in the figure.

9. [10 points] Draw a DPDA which accepts the language $\{a^n b^n \ : n \geq 0\}$ Once again, λ transitions are not allowed. Also, the input string ends with an eof marker, such as \$.