CSC 456/656 Fall 2025 Third Examination November 25, 2025

The entire exam is 310 points.

Name:____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

1. True/False/Open. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (i) **F** Every subset of a regular language is regular.

   (ii) **F** For any regular language L, there is a unique minimal NFA which accepts L.

   (iii) **T** The regular expressions $(a+b)^*$ and $(a+ab+b^*)^*$ are equivalent.

   (iv) **F** Every language is either $\mathcal{RE}$ or co–$\mathcal{RE}$.

   (v) **T** Every uncountable language is decidable. (By vacuous implication–there are no uncountable languages.)

   (vi) **O** $\mathcal{NC} = \mathcal{P}$–*time*.

   (vii) **O** $\mathcal{NP} = \mathcal{P}$–*time*.

   (viii) **O** $\mathcal{P}$–space $= \mathcal{P}$–*time*.

   (ix) **F** The set of all binary languages is countable.

   (x) **F** $\mathbb{R}$, the set of real numbers, is countable.

   (xi) **T** $\mathbb{Q}$, the set of rational numbers, is countable.

   (xii) **T** The complement of any decidable language is decidable.

   (xiii) **T** If S is any set, $|S| < \left|2^S\right|$.

   (xiv) **T** There are countably many recursive real numbers.

   (xv) **F** 2-SAT is known to be $\mathcal{NP}$–COMPLETE.

2. [10 points] Give a definition of the class $\mathcal{NC}$. A language L is $\mathcal{NP}$ if and only if there is some nondeterministic machine which accepts L in polynomial time.

3. [10 points] Give a definition of the class $\mathcal{P}$–complete. A language L is $\mathcal{P}$–complete if and only if there is an $\mathcal{NP}$ reduction of any $\mathcal{P}$–TIME language to L.

4. [20 points] Give a polynomial time reduction from 3-SAT to the Independent set problem.

   Let $E = C_1 * C_2 * \cdots * C_K$ be Boolean expression in 3-CNF form. Let $C_i = (t_{i,1} + t_{i,2} + t_{i,3})$, where each $t_{i,j}$ is a variable or the negation of a variable. Let $R(E)$ be the graph G_E where the vertices of G_E are $\{v_{i,j}\}$ for $1 \le i \le K$ and $j \in \{1,2,3\}$. There is an edge between distinct vertices $v_{i,j}$ and $v_{i',j'}$ if $i = i'$ or $v_{i,j} * v_{i',j'}$ is a contradiction.

5. [20 points] Prove that every recursively enumerable language is accepted by some machine.

Let L be L be an $\mathcal{RE}$ language, and let $w_1, w_2, \ldots$ be a recursive enumeration of L.

Let M be a machine which implements the following program.

```
read a string w
for all i
    if w = w_i
        accept w
```

Then M accepts L.

6. [20 points] Prove that every decidable language is enumerated in canonical order by some machine.

Let L be a decidable language, over an alphabet Σ. Let $w_1, w_2, \ldots$ be the canonical enumeration of Σ^*. The following program implements a machine M which enumerates L in canonical order.

```
for all i
    if w_i ∈ L
        write w_i
```

Then M enumerates L in canonical order.

7. [20 points] Prove that the halting problem is undecidable.

Let Q be a machine which implements the following program.

```
read a machine description ⟨M⟩.
if M halts with input ⟨M⟩
    run forever
else
    halt
```

Now run Q with input $\langle Q \rangle$. If it halts, that means that Q does not halt with input $\langle Q \rangle$, contradiction. But if it doesn't halt, that means that Q halts with input $\langle Q \rangle$, also a contraction. Thus the halting problem is undecidable.

8. [10 points] Give an example of a language L (or a 0-1 problem) which is

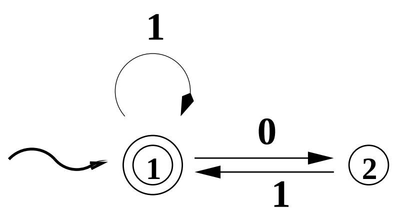
1. $\mathcal{NP}$.
2. co-$\mathcal{NP}$.
3. Not known to be $\mathcal{NP}$-complete.
4. Not known to be $\mathcal{P}$–TIME.

Here are two examples:

1. The graph isomorphism problem. Given two graphs G_1 and G_2, does there exist a 1-1 correspondence F between the vertices of the graphs which also gives a 1-1 correspondence between the edges of the graphs?

2. The binary integer factoring program. Given two binary numerals $\langle n \rangle$ and $\langle m \rangle$, where $m < n$, does n have an integer divisor d such that $2 \leq d \leq m$?

9. [10 points] Let L be the language consisting of all binary strings such that every 0 is followed by 1. Draw a DFA that accepts L.



10. [5 points] Write a regular expression for the language L given in Problem 9.

$$(1 + 01)^*$$

11. [10 points] If anyone finds a $\mathcal{P}$–TIME algorithm for factoring integers, what would be the real-life consequences?

Messages encrypted with the RSA algorithm could be deciphered in polynomial time.

12. [10 points] If anyone finds an $\mathcal{NC}$–algorithm for the circuit value problem, what would be the real-life consequences?

Any polynomial time problem could be solved in polylogarithmic time by polynomially many processors working in parallel.

13. [5 points] Give an example of a language which is generated by an unambiguous context-free grammar, but cannot be accepted by any DPDA.

There are many examples. One is the language of all palindromes over an alphabet of size 2.

14. [5 points] Give an example of an inherently ambiguous language.

There are many examples. One is the language $\left\{a^i b^j c^k \ : \ i = j \text{ or } j = k\right\}$

15. [10 points] What is a CNF (Chomsky Normal Form) grammar?

A context-free grammar with such that

1. There may be a production $S \to \lambda$, where S is the start symbol.
2. The right side of every other production is either two variables or one terminal.
3. If there is a production $S \to \lambda$, the start symbol may not occur on the right side of any production.

16. Every language falls into exactly one of the classes listed below.

A. Known to be $\mathcal{NC}$

B. Known to be $\mathcal{P}$, but not known to be $\mathcal{NC}$, and not known to be $\mathcal{NP}$-complete.

C. Known to be $\mathcal{NP}$, but not known to be $\mathcal{P}$, and not known to be $\mathcal{NP}$–complete.

D. Known to be $\mathcal{NP}$–comlete.

E. Known to be $\mathcal{P}$–SPACE, but not known to be $\mathcal{NP}$.

F. Known to be decidable, but not known to be $\mathcal{P}$–SPACE.

G. Known to be $\mathcal{RE}$, but not known to be decidable.

H. Known to be co-$\mathcal{RE}$, but not known to be decidable.

I. Not known to be $\mathcal{RE}$ and not known to be co-$\mathcal{RE}$.

For each of these languages, indicate which of the classes listed above it belongs to.

(i) [5 points] **D**. 3-SAT.

(ii) [5 points] **A**. Matrix multiplication.

(iii) [5 points] **B**. CVP, the circuit value problem.

(iv) [5 points] **D**. The jigsaw problem. Given a set of flat shapes, can you assemble them into a rectangle?

(v) [5 points] **E**. RUSH HOUR. Given a position, is there a way to get the red car out of the parking lot?

(vi) [5 points] **D**. TSP, the traveling salesman problem.

(vii) [5 points] **C**. The graph isomorphism problem. Is there a 1-1 correspondence of the vertices of a given graph G_1 to the vertices of another given graph G_2 which preserves edges? edges?

(viii) [5 points] **G**. The halting problem.

(ix) [5 points] **H**. The context-free grammar equivalence problem.

17. Consider the following context-free grammar G and LALR parser for G.

1. $E \to E +_2 E_3$
2. $E \to E *_4 E_5$
3. $E \to E \wedge_6 E_7$
4. $E \to x_8$

	x	$+$	$*$	$\wedge$	\$	E
0	$s8$					1
1		$s2$	$s4$	$s6$	HALT	
2	$s8$					3
3		$r1$	$s4$	$s6$	$r1$	
4	$s8$					5
5		$r2$	$r2$	$s6$	$r2$	
6	$s8$					7
7		$r3$	$r3$	$s6$	$r3$	
8		$r4$	$r4$	$r4$	$r4$	

(i) [5 points] Which entry in the action table guarantees that "+" is left-associative? Row 3, Col "+".

(ii) [5 points] Which entry in the action table guarantees that "*" is left-associative? Row 5, Col "$*$".

(iii) [5 points] Which entry in the action table guarantees that "$\wedge$" is right-associative? Row 7, Col "$\wedge$".

18. [10 points] State the pumping lemma for context-free languages.

For any context-free language L
There exists an integer p such that
For any $w \in L$ of length at least p
There exist strings u, v, x, y, and z such that
1. $w = uvxyz$ and
2. $|vxy| \leq p$ and
3. $|v| + |y| > 0$ and
4. For any integer $i \geq 0$ $uv^ixy^iz \in L$.