CSC 456/656 Fall 2025 First Examination September 18, 2025

The entire exam is 230 points.

Name:_____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

In the questions of this test, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.
If L is a language over an alphabet Σ, we define the *complement* of L to be the set of all strings over Σ which are not in L. If $\mathcal{C}$ is a class of languages, we define co-$\mathcal{C}$ to be the class of all complements of members of $\mathcal{C}$.

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   (i) _____ Every subset of a regular language is regular.

   (ii) _____ The complement of a regular language is always a regular language.

   (iii) _____ The class of regular languages is closed under union.

   (iv) _____ The Kleene closure of any CFL is a CFL.

   (v) _____ The class of regular languages is closed under intersection.

   (vi) _____ The class of context-free languages is closed under union.

   (vii) _____ The class of context-free languages is closed under intersection.

   (viii) _____ $\{a^n b^n : n \geq 0\}$ is regular.

   (ix) _____ $\mathcal{P} = \mathcal{NP}$.

   (x) _____ The complement of any $\mathcal{P}$–SPACE language is $\mathcal{P}$–SPACE.

   (xi) _____ If L is both $\mathcal{NP}$ and co–$\mathcal{NP}$, then L must be $\mathcal{P}$–TIME.

   (xii) _____ Given a regular language L, there is a unique minimal NFA which accepts L.

   (xiii) _____ If L_1 is a $\mathcal{NP}$-complete language and there is a $\mathcal{P}$–TIME reduction of L_1 to L_2, then L_2 must be $\mathcal{NP}$-hard.

   (xiv) _____ If L_2 is a $\mathcal{NP}$-complete language and there is a $\mathcal{P}$–TIME reduction of L_1 to L_2, then L_1 must be $\mathcal{NP}$-hard.

   (xv) _____ The Kleene closure of any regular language is regular.

   (xvi) _____ co-$\mathcal{P} = \mathcal{P}$.

   (xvii) _____ co-$\mathcal{NP} = \mathcal{NP}$.

   (xviii) _____ Block placement problems, such as the jigsaw puzzle problem, are $\mathcal{NP}$.

   (xix) _____ Sliding block problems, such as **Rush Hour** and the furniture mover's problem, are $\mathcal{P}$–SPACE.

(xx) _____ $\mathcal{P}$–SPACE $= \mathcal{NP}$

(xxi) _____ Every regular language is context-free.

(xxii) _____ The programming language C++ is context-free.

(xxiii) _____ The complement of any context-free language is context-free.

(xxiv) _____ Given any regular grammar G with n variables, there exists an NFA with n states that accepts $L(G)$.

(xxv) _____ Given an integer n, it is possible to find the prime factors of in $O(\log^k n)$ time for some constant k.

(xxvi) _____ Given an integer n, it is possible to decide whether n is prime in $O(\log^k n)$ time for some constant k.

(xxvii) _____ Every context-free language is accepted by some DPDA.

(xxviii) _____ Any language consisting of all decimal numerals of an arithmetic sequence (for example: $L = \{\langle 5 + 8n \rangle : n \geq 0\} = \{5, 13, 21, 29, 37, 45 \ldots\}$) is regular. Note: the members of L are numerals, not numbers.

(xxix) _____ Let L_1 be a regular binary language. Let L_2 be the language of all strings obtained from members of L_1 by substituting ab for 0 and c for 1. Then L_2 must be regular. For example, if $L_1 = \{0, 10, 10011\}$ then $L_2 = \{ab, cab, cababcc\}$.

2. [10 points] Give two context-free languages whose intersection is not context-free.

3. [10 points] Give a context-free grammar for the language $\{a^n b^m c^n : n \geq 0, m \geq 0\}$

4. [5 points] Let L be the set of binary numerals for positive integers which are equivalent to 1 modulo 3. (n such that $n\%3 = 1$) That is, $L = \{1, 100, 111, 1010, 1101, \ldots\}$. Draw a DFA that accepts L.

5. [5 points] What is $\emptyset^*$, the Kleene closure of the empty language?

6. [5 points] Draw a DFA which accepts $\{b^i a b^j : i, j \geq 0\}$, the language of all strings over $\{a, b\}$ which contain exactly one a.

7. For each regular expression below, draw a DFA which accepts the language desribed by that regular expression. (5 points each)

(i) λ

(ii) a^*

(iii) $\emptyset$

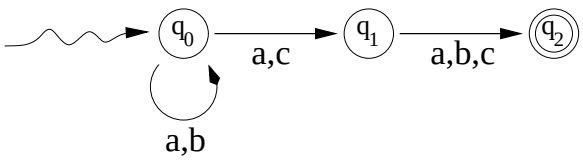
(iv) a^*ba^*

(v) $(ab)^*$

(vi) a^*b^*

(vii) $(a+b)^*$

8. [10 points] Draw a minimal DFA equivalent to the following three state NFA.



9. [10 points] Draw a DPDA which accepts the language $\{a^n b^n : n \geq 0\}$