CSC 456/656 Fall 2023 First Examination February 12, 2025

Name:_____

The entire test is 245 points.

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

In the questions of this test, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.
If L is a language over an alphabet Σ, we define the *complement* of L to be the set of all strings over Σ which are not in L. If $\mathcal{C}$ is a class of languages, we define co-$\mathcal{C}$ to be the class of all complements of members of $\mathcal{C}$.

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   (i) _____ Every subset of a regular language is regular.

   (ii) _____ Every language which has a mathematical definition is regular.

   (iii) The set of binary numerals for prime numbers is in $\mathcal{P}$–TIME.

   (iv) _____ Every language is countable.

   (v) _____ The set of languages over the binary alphabet is countable.

   (vi) _____ $\mathcal{P} = \mathcal{NP}$.

   (vii) _____ The complement of any $\mathcal{P}$–TIME language is $\mathcal{P}$–TIME.

   (viii) _____ The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.

   (ix) _____ Every finite language is regular.

   (x) _____ A language is regular if and only if it is accepted by some DFA.

   (xi) _____ A language is regular if and only if it is accepted by some NFA.

   (xii) _____ A language is regular if and only if it is generated by some regular grammar.

   (xiii) _____ The programming language C++ is regular.

   (xiv) _____ The union of any two regular languages is regular.

   (xv) _____ The intersection of any two regular languages is regular.

   (xvi) _____ The concatenation of any two regular languages is regular.

   (xvii) _____ The Kleene closure of any regular language is regular.

   (xix) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

(xx) _____ Every problem that can be mathematically defined has an algorithmic solution.

(xxi) _____ The intersection of any two $\mathcal{NP}$ languages is $\mathcal{NP}$.

(xxii) _____ $\mathcal{NP} = \mathcal{P}$-SPACE

2. [10 points] Suppose L is a problem such that you can check any suggested solution in polynomial time. Which one of these statements is certainly true?

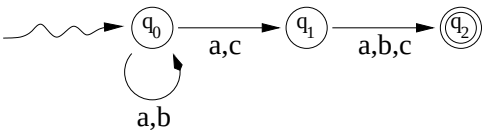
   (i) L is $\mathcal{P}$.

   (ii) L is $\mathcal{NP}$.

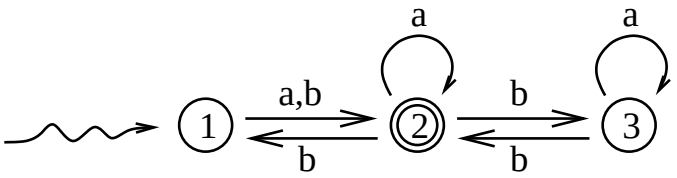
   (iii) L is $\mathcal{NP}$-complete.

3. [20 points] L be the language of all binary strings in which each 0 is followed by 1. Draw a DFA which accepts L.

4. [20 points] Give a grammar, with at most 3 variables, for the language accepted by the follwing NFA.



5. [20 points] Give a regular expression for the language accepted by the following NFA



6. [10 points] Give an example of a language which is not regular.

7. [20 points] Give a definition of the class $\mathcal{P}$–TIME.

8. [20 points] Let L be the language of all binary strings encoding numbers which are equivalent to 1 modulo 3, where leading zeros are allowed. Thus, $L = \{1, 01, 001, 100, 111, 0100, 0111, 1010, \ldots\}$. Draw a DFA which accepts L. (You need only three states.)

9. [20 points] Draw a minimal DFA equivalent to the NFA shown in the figure below. Show the transition table, and also show the matrix used for minimizing the DFA.

