CSC 456/656 Fall 2026 First Examination October 8, 2026

The entire exam is 350 points.

Name:_____

No books, notes, scratch paper, or machines. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

I left my "mind reading" cap at home. Therefore, if you want me to grade anything written on scratch paper, write your name and "Plase grade this page," and which problem it is, on that scratch paper. If you do not want me to grade a piece of scratch paper, do **not** hand it in. If you write a solution on the back of a page, write "See back of page 5 (or whatever)" next to the problem.

In the questions of this test, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.
If L is a language over an alphabet Σ, we define the *complement* of L to be the set of all strings over Σ which are not in L. If $\mathcal{C}$ is a class of languages, we define co-$\mathcal{C}$ to be the class of all complements of members of $\mathcal{C}$.

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   (i) **T** The language of binary numerals for multiples of 23 is regular.

   (ii) **T** Every language is countable.

   (iii) **T** The set of languages over the binary alphabet is uncountable.

   (iv) **O** $\mathcal{P} = \mathcal{NP}$.

   (v) **T** The complement of any $\mathcal{P}$–TIME language is $\mathcal{P}$–TIME.

   (vi) **O** The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.

   (vii) **T** The complement of any decidable language is decidable.

   (viii) **T** The complement of any undecidable language is undecidable.

   (ix) **O** $\mathcal{NP} = \mathcal{P}$–SPACE.

   (xi) **T** The complement of any $\mathcal{P}$–SPACE language is $\mathcal{P}$–SPACE.

   (xii) **T** The complement of every recursive language is recursive.

   (xiii) **T** Given any unambiguous context-free grammar G and any string $w \in L(G)$, there is always a unique leftmost derivation of w using G.

   (xiv) **F** For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.

   (xv) **T** If $\mathcal{P} = \mathcal{NP}$, then all one-way encoding systems are breakable in polynomial time.

(xvi) **T** A language L is in $\mathcal{NP}$ if and only if there is a polynomial time reduction of L to SAT.

(xvii) **T** The intersection of any context-free language with any regular language is context-free.

(xviii) **T** If L_1 reduces to L_2 in polynomial time, and if L_2 is $\mathcal{NP}$ and L_1 is $\mathcal{NP}$-complete, then L_2 must be $\mathcal{NP}$-complete.

(xix) **F** Every function that can be mathematically defined is recursive (computable).

(xx) **T** The halting problem undecidable.

(xxi) **F** If there is a recursive reduction of a language L to the halting problem, then L must be undecidable.

(xxii) **T** The set of rational numbers is countable.

(xxiii) **F** The set of real numbers is countable.

(xxiv) **O** The factoring problem for unary numerals is $\mathcal{P}$–TIME

(xxv) **F** The set of all positive real numbers is countable.

(xxvi) **T** If L is a context-free language over the unary alphabet, then L must be regular.

(xxvii) **T** co-$\mathcal{P}$–TIME = $\mathcal{P}$–TIME

(xxviii) **F** Every subset of a regular language is regular.

(xxix) **F** The complement of a CFL (context-free language) is always a CFL.

(xxx) The class of context-free languages is closed under union.

(xxxi) **F** The class of context-free languages is closed under intersection.

(xxxii) The set of binary numerals for prime numbers is $\mathcal{P}$–TIME.

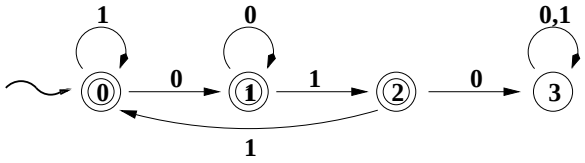
(xxxiii) **F** The set of languages over the binary alphabet is countable.

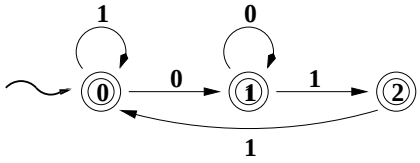
(xxxix) **O** If L is both $\mathcal{NP}$ and co–$\mathcal{NP}$, then L must be $\mathcal{P}$–TIME.

(xl) **T** If L is both $\mathcal{RE}$ and co–$\mathcal{RE}$, then L must be decidable.

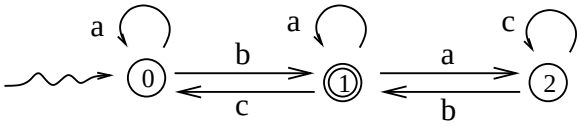
(xli) **T** Every context-free language is $\mathcal{P}$–TIME.

2. [20 points] L be the language of all binary strings which do not have the substring 010. Draw a DFA which accepts L.



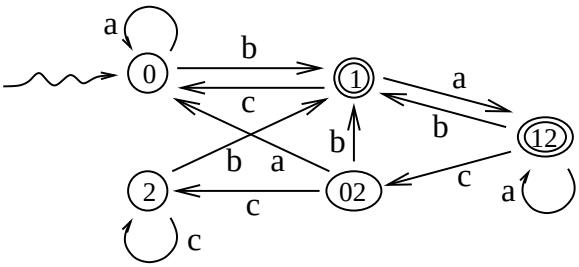


3. [20 points] Consider the NFA M pictured below.



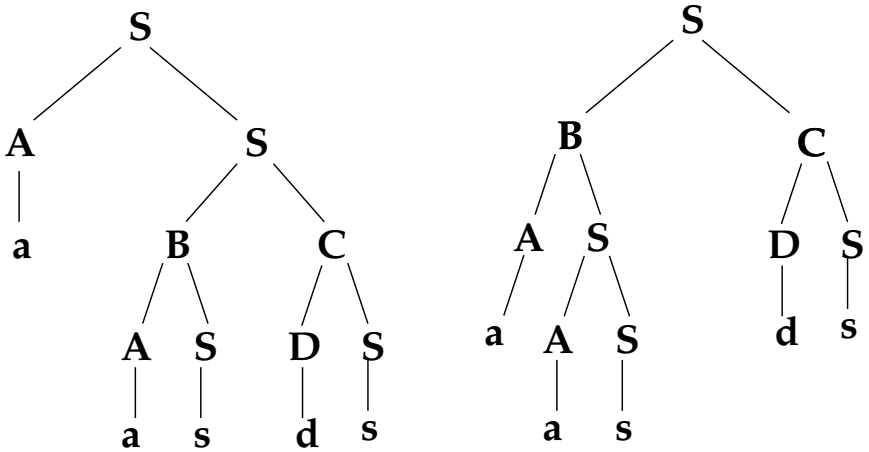
Construct a minimal DFA equivalent to M.

	a	b	c
0	0	1	$\emptyset$
1	12	$\emptyset$	0
2	$\emptyset$	1	2
12	12	1	02
02	0	1	2



4. [20 points] Let G be the CF grammar given below. Prove that G is ambiguous by giving two different parse trees for the string *aasds*.

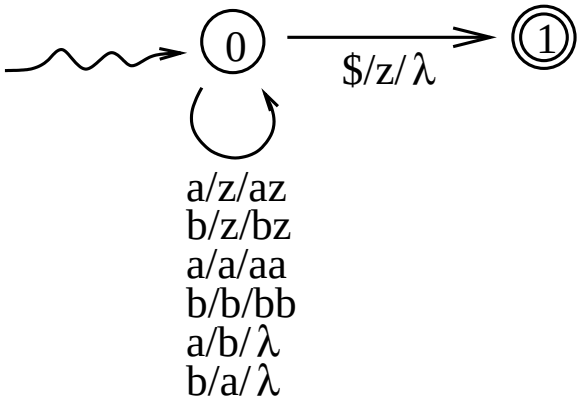
1. $S \to s$
2. $S \to AS$
3. $A \to a$
4. $S \to BC$
5. $B \to AS$
6. $C \to DS$
7. $D \to d$



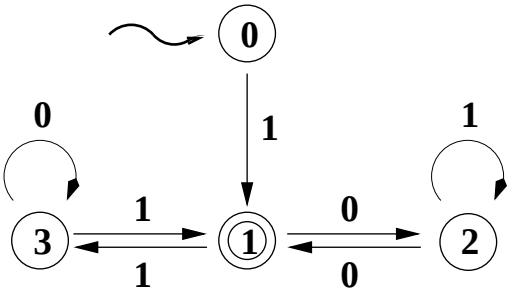
5. [20 points] Let L be the language consisting of all strings over $\{a, b\}$ which have equal numbers of each symbol. Give a CFG for L.

$S \to aSbS$
$S \to aSbS$
$S \to \lambda$

6. [20 points] Draw a PDA which accepts the language given in problem 5.



7. [20 points] Let L be the set of binary numerals for positive integers which are equivalent to 1 modulo 3. (n such that $n\%3 = 1$) That is, $L = \{1, 100, 111, 1010, 1101, \ldots\}$. Draw a DFA that accepts L.



1. [20 points] State the pumping lemma for regular languages.

For any regular language L, there exists an integer p such that, for any $w \in L$ of length at least p there exist strings x, y, z such that

1. $w = xyz$
2. $|xy| \leq p$
3. $y \neq \lambda$
4. For any $i \geq 0$ $xy^iz \in L$

2. [20 points] Write the "certificate-verification" definition of the class $\mathcal{NP}$.

3. [20 points] Give a polynomial time reduction of the subset sum problem to partition.