**University of Nevada, Las Vegas**
**Computer Science 456/656 Spring 2005**
**Final Examination**

The entire examination is 225 points.

1. True or False. [5 points each]

   (a) _____ Every subset of a regular language is regular.

   (b) _____ The complement of every recursive language is recursive.

   (c) _____ The complement of every recursively enumerable language is recursively enumerable.

   (d) _____ Every language which is generated by an unrestricted grammar is recursively enumerable.

   (e) _____ The question of whether two context-free grammars generate the same language is undecidable.

   (f) _____ If $L_1$ reduces to $L_2$ recursively, and if $L_1$ is undecidable, then $L_2$ must be undecidable.

   (g) _____ The question of whether two regular expressions are equivalent is decidable.

   (h) _____ The complement of every recursively enumerable language is recursively enumerable.

   (i) _____ Let $L = \{\langle M \rangle \mid M \text{ halts with no input}\}$. Then $L$ is recursively enumerable.

   (j) _____ An undecidable language is necessarily $\mathcal{NP}$-complete.

   (k) _____ The language of all binary strings which are the binary numerals for prime numbers is context-free.

2. [5 points] If there is an easy reduction from $L_1$ to $L_2$, then _____ is at least as hard as _____.

3. [15 points] Let $L$ be the language of all binary numerals for positive integers equivalent to 2 modulo 3. Thus, for example, the binary numerals for 2, 5, 8, 11, 14, ... are in $L$. We allow a binary numeral to have leading zeros; thus (for example) $001110 \in L$, since it is a binary numeral for 14. Draw a minimal DFA which accepts $L$.

4. [5 points] What class of machines accepts the class of context free languages? _____

5. [5 points] What class of machines accepts the class of regular languages? _____

6. [5 points] What class of machines accepts the class of recursively enumerable languages? _____

7. [10 points] Give two examples of languages or problems which are $\mathcal{NP}$-complete. (Do not attempt to prove that they are $\mathcal{NP}$-complete. You may refer to a language or problem by its standard name, such as, "The boolean satisfiability problem.")

8. [20 points] Define what it means to say that a certain language $L$ is *accepted in polynomial time* by a certain machine $M$.

9. [20 points] Give a definition of *$\mathcal{NP}$-complete language.*

10. [25 points] Consider the language $L$ of all strings which would be acceptable as algebraic expressions involving variables and constants, where:

- Every variable name is either $x$, $y$, or $z$.
- Every constant is a natural number between 0 and 9
- The only operators are addition, subtraction, and multiplication.
- The symbol '$-$' is used only for subtraction. There is no negation.
- There is no multiplication symbol. Multiplication is indicated by concatenating strings.
- In multiplication of a constant by anything else, the constant must come first, and there can be at most one constant factor in any term.
- Parentheses can be used.

Here are some strings in the language $x(y + 2z)$, $x - 1 - z$, $4(zx - 2y)(x + z(x - 1))$.

Give a grammar for $L$ which is consistent with the usual semantics (as you learned in school) of such expressions.

11. [30 points] Let $L$ be the set of all strings over $\{a, b\}$ which have equal numbers of $a$'s and $b$'s. Prove, by contradiction, that $L$ is not regular, as follows. First, you know that $\{a^n b^n\}$ is not regular (don't prove it; you can just take it as given). Second, you know that $\{a^i b^j\}$ is regular. (How do you know this?) Third, there is a theorem about intersections of regular language. Finally, obtain a contradiction.

12. [30 points] Consider the following context-free grammar $G$:

$S \to a$

$S \to SbS$

$S \to SS$

Prove that $w = abaabaab$ is, or is not, generated by $G$, as follows:

(a) Find a context-free grammar $G'$ equivalent to $G$ such that no production has more than 2 symbols on the right hand side.

(b) Use the CYK algorithm to determine whether $w$ is generated by $G'$, using Figure 1.

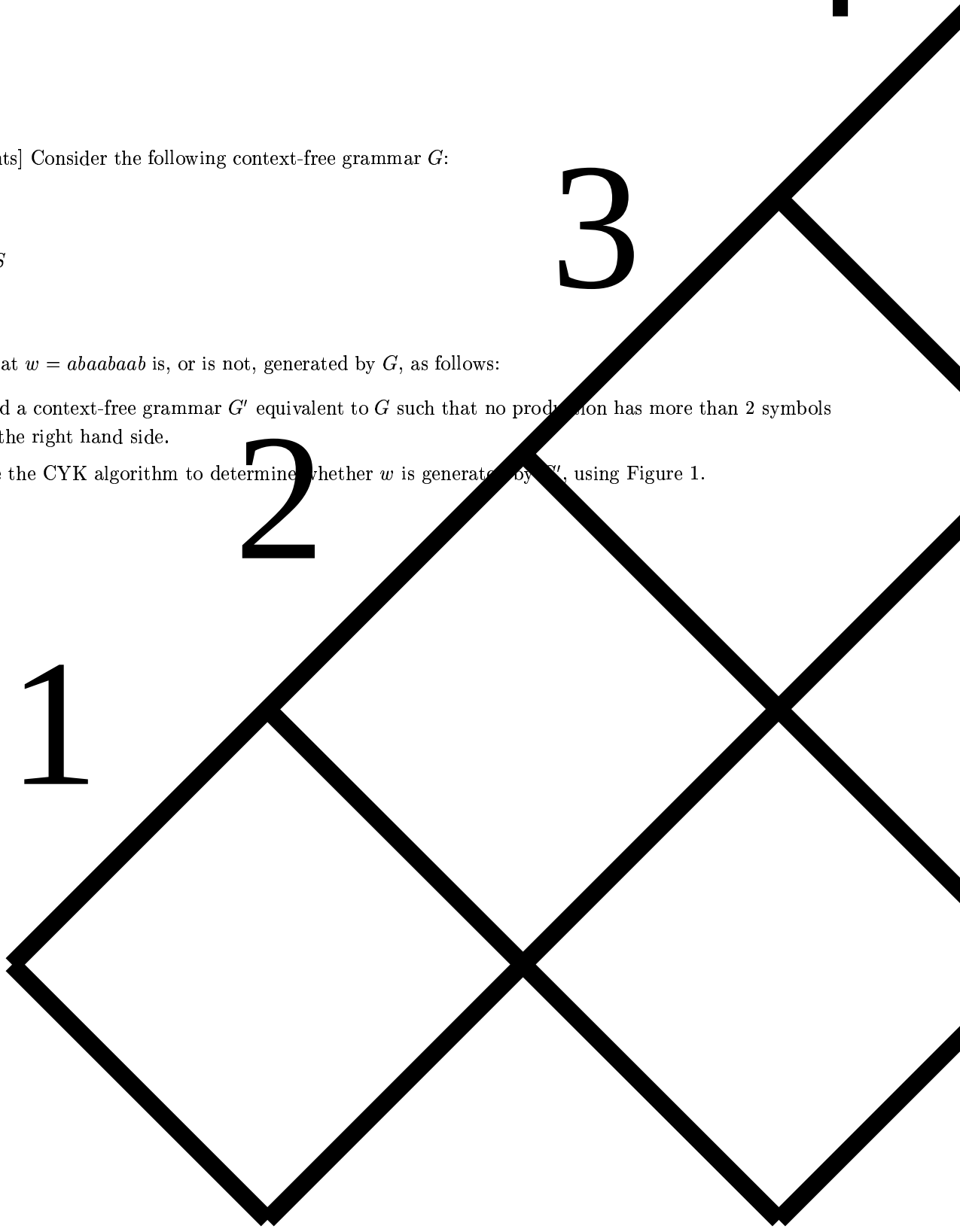



Figure 1: Fill in the cells to work problem 12.