Computer Science 456/656 Spring 2018 Practice Examination March 21, 2018

The entire examination is 690 points.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (Review the 3 page True/False homework given for February 28 and March 5.)

   (a) _____ Every subset of a regular language is regular.

   (b) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m, n \geq 0$. Then L is a regular language.

   (c) _____ The complement of every regular language is regular.

   (d) _____ The Kleene closure of every context-free language is context-free.

   (e) _____ If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

   (f) _____ If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

   (g) _____ There is a PDA that accepts all valid C++ programs.

   (h) _____ The intersection of any two regular languages is regular.

   (i) _____ The language consisting of all base 7 numerals for positive integers n such that $n \% 3 = 2$ is regular.

   (j) _____ The intersection of any two context-free languages is context-free.

   (k) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n c^m$, where $m, n \geq 0$. Then L is a context-free language.

   (l) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then L is a context-free language.

   (m) _____ The complement of every context-free language is context-free.

   (n) _____ The union of any two context-free languages is context-free.

   (o) _____ If a language has an context-free grammar, then it is is accepted by some push-down automaton.

   (p) _____ Every context-free language has an unambiguous context-free grammar.

   (q) _____ Every language that has an unambiguous context-free grammar is accepted by some DPDA.

   (r) _____ Every deterministic machine is a non-deterministic machine.

   (s) _____ The language consisting of all base 2 numerals for integer powers of 2 is regular.

   (t) _____ There is a DPDA that accepts the language of all palindromes over the binary alphabet $\{0, 1\}$.

   (a) _____ There exists a machine[1] that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

[1] As always in automata theory, "machine" means abstract machine, a mathematical object whose memory and running time are **not** constrained by the size and lifetime of the known (or unknown) universe, or any other physical laws. If we want to discuss the kind of machine that exists (or could exist) physically, we call it a "physical machine."

(b) _____ For every real number x, there exists a machine that runs forever and outputs the string of binary digits of x.

(c) _____ If a language has an unambiguous context-free grammar, then there must be some DPDA that accepts it.

(d) _____ The problem of whether a given string is generated by a given context-free grammar is $\mathcal{P}$-TIME.

(e) _____ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

(f) _____ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

(g) _____ There exists a mathematical proposition that can be neither proved nor disproved.

(h) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$.

(i) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

(j) _____ The problem of whether a given context-free grammar generates all strings is decidable.

(k) _____ The language $\left\{a^i b^j c^k \mid j \geq i + k\right\}$ is context-free.

(l) _____ If a language L is undecidable, then there can be no machine that enumerates L.

(m) Recall that if $\mathcal{L}$ is a class of languages, co-$\mathcal{L}$ is defined to be the class of all languages that are not in $\mathcal{L}$.

_____ Let $\mathcal{RE}$ be the class of all recursively enumerable languages. If L is in $\mathcal{RE}$ and also L is in co-$\mathcal{RE}$, then L must be decidable.

(n) _____ If a language has an unambiguous context-free grammar, then there must be some DPDA that accepts it.

(o) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a unary numeral.

(p) _____ Every bounded function is recursive.

(q) _____ For any two languages L_1 and L_2, if L_1 is $\mathcal{NP}$-complete, L_2 is $\mathcal{NP}$, and there is a polynomial time reduction of L_2 to L_1, then L_2 must be $\mathcal{NP}$-complete.

(r) _____ If P is a mathematical proposition that can be stated using n binary bits, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

Fill in the blanks. [5 points each blank]

(a) An LALR _____ outputs a _____ derivation.

(b) An _____ of a language L is a machine that outputs all the strings of L and no other strings.

2. [20 points] State the pumping lemma for regular languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

3. [30 points] State the Church-Turing thesis, and explain (in about 5 lines or less) why it is important.

4. [30 points] Give a sketch of the proof that the independent set problem is $\mathcal{NP}$-complete, assuming that 3-CNF-SAT is $\mathcal{NP}$-complete. You may draw pictures and use examples.

5. [30 points] What is the diagonal language? Give a brief sketch of the proof that it is not accepted by any Turing machine. (If you use more than this page, you are writing too much.)

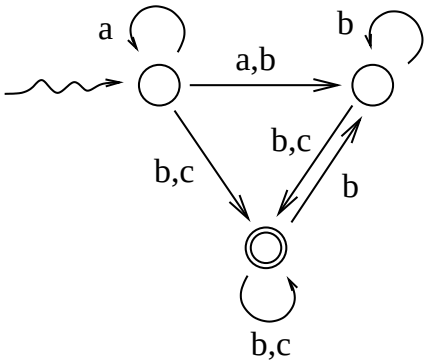
6. [20 points] Describe, in English, what the Turing machine diagrammed below does. Hint: It only takes a few words.

   (Look at one of the sample problems from the Assignments page.)

7. [30 points] Prove that, if a language L is decidable, then L can be enumerated in canonical order by some machine.

8. [25 points] Draw an NFA with five states which accepts the language described by the regular expression $(0+1)^*0(0+1)(0+1)(0+1)$

9. [25 points] Write a regular expression for the language accepted by the following NFA. If your answer is unnecessarily long by a wide margin, I might mark it wrong even if it's right.



Find a Regular Expression

10. [20 points] Let G be the context-free grammar given below.

    (a) $S \to a$

    (b) $S \to wS$

    (c) $S \to iS$

    (d) $S \to iSeS$

    Prove that G is ambiguous by writing two different **leftmost** derivations for the string *iwiaea*. [If you simply show two different parse trees, you are not following instructions.]

11. [30 points] Design a PDA that accepts the language $L = \{a^n bc^n \ : \ n \geq 0\}$.

12. [30 points] Give a context-free grammar for the language of all strings over $\{0, 1\}$ of the form $0^m 1^n$ where $n \neq m$.

13. [30 points] The following context-free grammar G is ambiguous. Give an equivalent unambiguous grammar.

14. Fill in the ACTION and GOTO tables for the ambiguous grammar given in Problem 13 (This problem is hard. I will give more clues in class.)

	a	b	c	$+$	$-$	$*$	**eof**	E
0								
1							**halt**	
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								

15. [30 points] Let L be the language generated by the Chomsky Normal Form (CNF) grammar given below.

(a) $S \to a$

(b) $E \to a$

(c) $S \to LA$

(d) $E \to LA$

(e) $L \to ($

(f) $A \to ER$

(g) $R \to)$

(h) $S \to PE$

(i) $E \to PE$

(j) $S \to EE$

(k) $E \to EE$

(l) $P \to EQ$

(m) $Q \to +$

Use the CYK algorithm to prove that the string $a(a + a)$ is a member of L.

16. [15 points] State the pumping lemma for regular languages.

17. Let L be the language generated by the context-free grammar given in the first box. The second box contains a Chomsky Normal Form (CNF) grammar that also generates L.

(a) $S \to a$

(b) $S \to wS$

(c) $S \to iS$

(d) $S \to iSeS$

(a) $S \to WS$

(b) $S \to IS$

(c) $S \to AB$

(d) $A \to IS$

(e) $B \to ES$

(f) $S \to a$

(g) $I \to i$

(h) $W \to w$

(i) $E \to e$

(a) [20 points] Use the CYK algorithm to prove that the string $iwiaea$ is a member of L.

(b) [20 points] By looking at your work carefully, you can determine that the CNF grammar given above is ambiguous. Write two different parse trees for $iwiaea$, using the CNF grammar.

18. [30 points] Consider the NFA whose transition diagram is in Figure 1 below. where the input alphabet is $\{a, b, c\}$. Draw the transition diagram of an equivalent minimal DFA. Show your steps.

19. [30 points] Let $L = \left\{w \in \{a, b\}^* \mid \#_a(w) = 2\#_b(w)\right\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w. For example, $aaababaaabba \in L$, because that string has the twice as many a's as b's. Give a context-free grammar for L. Your grammar may be ambiguous.

20. [30 points]

1. $S \to \epsilon$
2. $S \to a_2 S_3 b_4 S_5$

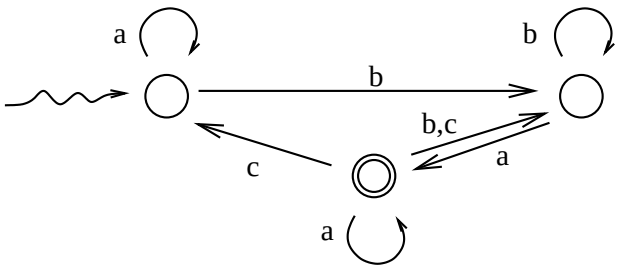


Figure 1: Find a minimal DFA equivalent to this NFA

	a	b	**eof**	S
0				
1			**halt**	
2				
3				
4				
5				

Complete the ACTION and GOTO tables of an LALR parser for the grammar given above. This grammar unambiguously generates the Dyck language, where (to avoid difficulty grading) a represents a left parenthesis, and b represents a right parenthesis. Example strings include ϵ, ab, $aabb$, $abab$, and $aabbab$.