

University of Nevada, Las Vegas Computer Science 456/656 Spring 2022

Assignment 4: Due Wednesday March 30 2022

Name: _____

You are permitted to work in groups, get help from others, read books, and use the internet. You will receive a message from our graduate assistant telling you how to turn in the assignment.

Throughout this assignment, you may assume that a language is recursively enumerable if and only if it is accepted by some machine. Recall that “ L is recursively enumerable (RE)” means that there is a machine that enumerates L .

1. True/False/Open

- (a) _____ Every subset of a regular language is regular.
- (b) _____ If L_1 is $\mathcal{NP}$ -complete and L_2 is $\mathcal{NP}$, there is a $\mathcal{P}$ -TIME reduction of L_1 to L_2 .
- (c) _____ If L_1 is $\mathcal{NP}$ -complete and L_2 is $\mathcal{NP}$ and there is a $\mathcal{P}$ -TIME reduction of L_1 to L_2 , then L_2 is $\mathcal{NP}$ -complete.
- (d) _____ If L is $\mathcal{NP}$ -complete, there is no polynomial time algorithm which decides L .
- (e) _____ Every $\mathcal{NP}$ language is decidable.
- (f) _____ $\mathcal{NP} = \text{co-}\mathcal{NP}$.
- (g) _____ If L_1 is undecidable and there is a recursive reduction of L_1 to L_2 , then L_2 is undecidable.
- (h) _____ The CF grammar equivalence problem is recursively enumerable.
- (i) _____ If a language L is decidable, then there must be a machine that enumerates L in canonical order.
- (j) _____ If there is a machine that enumerates a language L , then L must be decidable.
- (k) _____ If there is a machine that accepts a language L , then L must be recursively enumerable (RE).
- (l) _____ If a language L is decidable, there is a machine that enumerates L .
- (m) _____ If there is a machine that enumerates a language L in canonical order, then L must be decidable.
- (n) _____ If $f : \mathcal{N} \rightarrow \mathcal{N}$ is a one-to-one and onto function, where $\mathcal{N}$ is the natural numbers (positive integers) we define the *inverse* of f to be a function $g : \mathcal{N} \rightarrow \mathcal{N}$ such that $f(g(n)) = n$ and $g(f(n)) = n$ for all $n \in \mathcal{N}$. There exists a one-to-one onto function $f : \mathcal{N} \rightarrow \mathcal{N}$ which can be computed in polynomial time whose inverse cannot be computed in polynomial time. (Such a function is called a *one-way* function.)
- (o) _____ There exists a recursive function T such that, for any provable statement P , there is a proof of P whose length does not exceed $T(n)$, where n is the length of P .

2. Consider the following CF grammar and LALR parser.

1. $S \rightarrow i_2 S_3$ 2. $S \rightarrow i_2 S_3 e_4 S_5$ 3. $S \rightarrow w_6 S_7$ 4. $S \rightarrow a_8$	ACTION					GOTO	
	a	i	e	w	$\$$	S	
	0	$s8$	$s2$		$s6$		1
	1				halt		
	2	$s8$	$s2$		$s6$		3
	3			$s4$		$r1$	
	4	$s8$	$s2$		$s6$		5
	5			$r2$		$r2$	
	6	$s8$	$s2$		$s6$		7
	7			$r3$		$r3$	
	8			$r4$		$r4$	

Walk through the computation of this parser where the input string is $iiwaeia$.

- Let L be a decidable language. Write a program in pseudo-code that enumerates L in canonical order.
- Let $L = \{ \langle G_1 \rangle \langle G_2 \rangle : G_1, G_2 \text{ are CF grammars that are not equivalent} \}$. Prove that L is recursively enumerable. Assume that the terminal alphabet of both grammars is Σ .

5. Prove that the halting problem is undecidable.

6. Given that 3-SAT is $\mathcal{NP}$ -complete, prove, by reduction, that IND, the independent set problem, is also $\mathcal{NP}$ -complete.