University of Nevada, Las Vegas Computer Science 456/656 Spring 2022

Assignment 5: Due Wednesday April 13 2022

8:30 AM

Name:_____

You are permitted to work in groups, get help from others, read books, and use the internet. Do not turn this assignment in.

1. True/False/Open

   (a) _____ Every subset of a recursively enumerable langugage is recursively enumerable.

   (b) _____ If L_1 is $\mathcal{NP}$ and L_2 is $\mathcal{NP}$-complete, there is a $\mathcal{P}$-TIME reduction of L_1 to L_2.

   (c) _____ If L_1 is $\mathcal{NP}$-complete and L_2 is $\mathcal{NP}$ and there is a $\mathcal{P}$-TIME reduction of L_1 to L_2, then L_2 is $\mathcal{NP}$-complete.

   (d) _____ If L is $\mathcal{NP}$-complete, there is no polynomial time algorithm which decides L.

   (e) _____ Every $\mathcal{NP}$ language is decidable.

   (f) _____ $\mathcal{NP}$ = co-$\mathcal{NP}$.

   (g) _____ If L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 is undecidable.

   (h) _____ The CF grammar equivalence problem is RE.

   (i) _____ The CF grammar equivalence problem is co-RE.

   (j) _____ If a language L is decidable, then there must be a machine that enumerates L in canonical order.

   (k) _____ If a language L is decidable, the complement of L is decidable.

   (l) _____ If a language L is undecidable, the complement of L is undecidable.

   (m) _____ If a language L is recursively enumerable, the complement of L is recursively enumerable.

   (n) _____ If there is a machine that enumerates a language L, then L must be decidable.

   (o) _____ Every language has a canonical order enumeration.

   (p) _____ If there is a machine that accepts a language L, then L must be recursively enumerable (RE).

   (q) _____ If a language L is decidable, there is a machine that enumerates L.

   (r) _____ If a language L is decidable, there is a machine that enumerates L in canonical order.

   (s) _____ There exist infinitely many one-way functions.

   (t) _____ Every regular language is in $\mathcal{NC}$.

   (u) _____ Every context-free language is in $\mathcal{NC}$.

   (v) _____ The Boolean circuit probolem is in $\mathcal{NC}$.

2. Consider the following CF grammar and LALR parser for an algebraic language. In this language, unary $-$ has highest precedence, $\wedge$ has precendence over $*$, which has precedence over $+$ and binary $-$. Operators $+$, $-$, $*$ are left associative, and $\wedge$ is right associative.

1. $E \rightarrow E_{1,13} +_2 E_3$
2. $E \rightarrow E_{1,13} -_4 E_5$
3. $E \rightarrow E_{1,2,4,13} *_6 E_7$
4. $E \rightarrow E_{1,3,5,7,9} \wedge_8 E_9$
5. $E \rightarrow -_{10} E_{11}$
6. $E \rightarrow (_{12} E_{13})_{14}$
7. $E \rightarrow \mathbf{id}_{15}$

(a) Fill in the missing entries in the tables:
Row 2 columns **id**, $-$, (, E
Row 3 columns $+$, $-$, $*$
Row 5 columns $+$, $-$, $*$
Row 9 columns $*$, $\wedge$

(b) Walk through the computation of the parser, where the input string is $(u + v * x) \wedge y \wedge -z$

	ACTION								GOTO
	id	$+$	$-$	$*$	$\wedge$	(	)	\$	E
0	$s15$		$s10$			$s12$			1
1		$s2$	$s4$	$s6$	$s8$			halt	
2									
3		$r1$					$r1$	$r1$	
4	$s15$		$s10$			$s12$			5
5		$r2$					$r2$	$r2$	
6	$s15$		$s10$			$s12$			7
7		$r3$	$r3$	$r3$	$s8$		$r3$	$r3$	
8	$s15$		$s10$			$s12$			9
9		$r4$	$r4$				$r4$	$r4$	
10	$s15$		$s10$			$s12$			11
11		$r5$	$r5$	$r5$	$r5$		$r5$	$r5$	
12	$s15$		$s10$			$s12$			13
13		$s2$	$s4$	$s6$	$s8$		$s14$		
14		$r6$	$r6$		$r6$		$r6$	$r6$	
15		$r7$	$r7$		$r7$		$r7$	$r7$	

3. Which of these problems, or languages, are known to be $\mathcal{NP}$-complete?

   (a) The firehouse problem: given a graph G and a number K, does there exist a set F of vertices of G of cardinality K such that every vertex of G is either a member of F or adjacent to a member of F?

   (b) The **bounded** subset sum problem. Given an integer K and a list of n positive integers, $x_1, \ldots x_n$, such that $x_i \leq 100$ for each i, does there exist a sublist whose sum is exactly K?

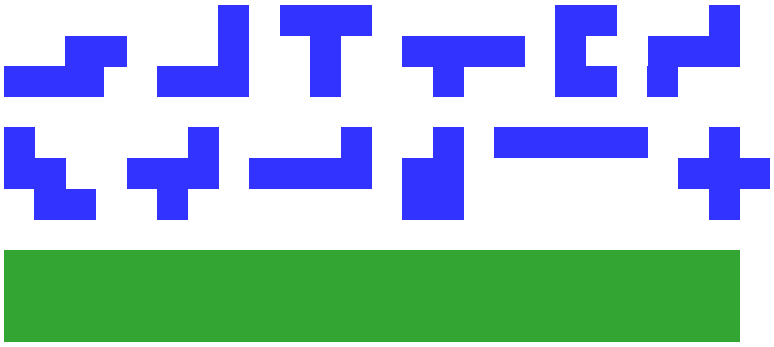
   (c) Boolean satisfiability.

   (d) The Boolean circuit problem.

   (e) 2-SAT, the set of all satisfiable Boolean expressions in 2-CNF form.

   (f) The set of all configurations of RUSH-HOUR, for any size board, from which it is possible to win.

   (g) The tiling problem: given a finite set of small polygons and one large polygon, is it possible to place all the small polygons so as to exactly cover the large polygon? Here is an example instance of this problem.



Since you're not turning this assignment in, I don't have to give you room to work these problems on this page.

4. State the pumping lemma for context-free languages.

5. Give a polynomial time reduction of the subset sum problem to the partition problem.

6. Give a polynomial time reduction of 3-SAT to the independent set problem.

7. Give a context-sensive grammar for the language $\{a^n b^n c^n d^n \ : \ n \geq 1\}$.

8. Give a context-sensitive grammar for the language $\{a^n \ : \ n \text{ is a power of } 2\}$

9. Suppose a machine M accepts a language L over an alphabet Σ. Prove that L is recursively enumerable.

10. Suppose a language L is recursively enumerable. Prove that L is accepted by some machine.

11. State the Church-Turing thesis, and explain why it is important.

12. Why the question of whether $\mathcal{P} = \mathcal{NC}$ important?

13. Prove that the halting problem is undecidable.