University of Nevada, Las Vegas Computer Science 456/656 Spring 2025

Answers to Assignment 5: Due Saturday March 8, 2025

If w is any string, we write w^{-1} for the *inverse* of w. For example, if $w = abcca$ then $w^{-1} = accba$. A *palindrome* is a string which is equal to its own inverse, such as "level." We write $\#_0(w)$ for the number of zeros in w, and $\#_1(w)$ for the number of ones.

1. There will be True/False questions on the second examination. Study the handout tf2.pdf.

2. Give a correct statement of the pumping lemma for regular languages. Be sure to keep track of quantifiers and conditionals. An answer which contains all the right words but which is not logically correct might get no credit.

   For any regular language L
   There exists a number p such that
   For any $w \in L$ of length at least p
   There exist strings x, y and z such that the following four statements hold:
   1. $w = xyz$
   2. $|xy| \leq p$
   3. $|y| \geq 1$
   4. **For any** $i \geq 0$ $xy^iz \in L$

3. Prove that the halting problem is undecidable.

   Assume that the halting problem is decidable. That means that for any program P and string w, the question of whether P halts with input w is decidable. Let H be a program which decides the halting problem. The input of H is P, w and its output is 0 or 1.

   Let Q be the program given below, whose input is any program P. Read P.
   If $H(P, P) = 1$ enter an infinite loop.
   Else halt.

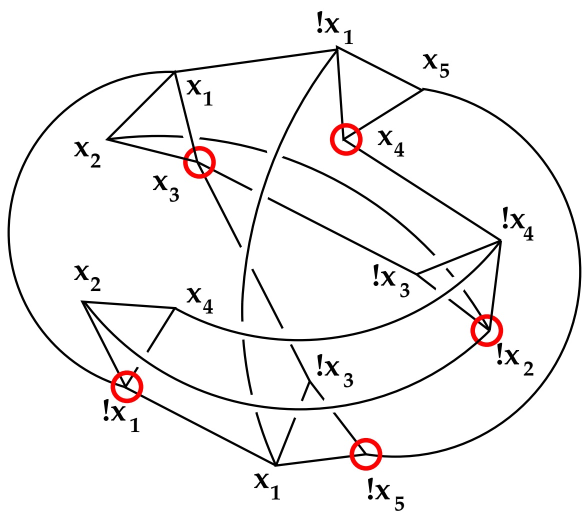
   Note that H must be a subprogram of Q. We now ask that happens if we run Q with input Q.

   If $H(Q, Q) = 1$, then Q enters an infinite loop, which means $H(Q, Q) = 0$.
   On the other hand, if $H(Q, Q) = 0$, then Q halts, which means $H(Q, Q) = 1$
   Contradiction. We conclude that Q cannot exist, hence H cannot exist, meaning that there is no algorithm which solves the halting problem.

4. Give a $\mathcal{P}$–TIME reduction of 3-SAT to IND, the independent set problem. Pictures might help.

   Given a Boolean expression e in 3-CNF form with K clauses, we construct a graph G which has an independent set of K vertices if and only if e is satisfiable. Let $t_{i,j}$ be the j^{th} term of the i^{th} clause of e. The vertices of G are $v_{i,j}$ for all $1 \le i \le K$ and $1 \le j \le 3$. There is an edge from $v_{i,j}$ to $v_{i',j'}$ if and only if either $i = i'$ and $j \ne j'$, or $i \ne i'$ and $t_{i,j} * t_{i',j'}$ is a contradiction.

   Example. Let $K = 5$, and let $e = (x_1 + x_2 + x_3) * (!x_1 + x_4 + x_5) * (!x_2 + !x_3 + !x_4) * (x_1 + !x_3 + !x_5) * (!x_1 + x_2 + x_4)$ In the figure, we label each vertex of G with the corrsponding term of e.



   We choose an independent set of vertices of G, each circled in red[1]. The corresponding satisfying assignment of e is
   $x_1 = 0$
   $x_2 = 0$
   $x_3 = 1$
   $x_4 = 1$
   $x_5 = 0$

5. Give a $\mathcal{P}$–TIME reduction of the subset sum problem to Partition.

   Let $A = (x_1, x_2, \ldots x_n, K)$ be any instance of the subset sum problem. Let $S = \sum_{i=1}^{n} x_i$. Let $B = (x_1, \ldots x_n, K+1, S-K+1)$, and instance of the partition problem. Then A has a solution if and only if B has a solution.

   The reason for the "+ 1" in the last two entries of B is that, without those terms, B has a solution regardless of whether A has a solution, since B can always be partitioned into $(x_1, \ldots x_n)$ and $(K, S-K)$.

[1]There are many choices of order 5 independent set for this example.