University of Nevada, Las Vegas Computer Science 456/656 Spring 2025

Assignment 4: Due Saturday March 1, 2025, 11:59 PM

Name:____

You are permitted to work in groups, get help from others, read books, and use the internet. You will receive a message from the graduage assistant, Louis, telling you how to turn in the assignment.

If w is any string, we write w^{-1} for the *inverse* of w. For example, if $w = abcca$ then $w^{-1} = accba$. A *palindrome* is a string which is equal to its own inverse, such as "level." We write $\#_0(w)$ for the number of zeros in w, and $\#_1(w)$ for the number of ones.

1. True or False. If the answwer is unknown to science at this time, write **O** for Open.

   (a) ________ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (b) ________ The union of any two context-free languages must be context-free

   (c) ________ The language $\{a^n b^n \mid n \geq 0\}$ is context-free.

   (d) ________ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

   (e) ________ The language $\left\{a^i b^j c^k \mid j = i + k\right\}$ is context-free.

   (f) ________ The intersection of any regular language with any context-free language is context-free.

   (g) ________ The intersection of any two context-free languages is context-free.

   (h) ________ If L is a context-free language over an alphabet with just one symbol, then L is regular.

   (i) ________ There is a deterministic parser for any context-free grammar.

   (j) ________ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.

   (k) ________ The Kleene closure of any context-free language is context-free.

   (l) ________ Every regular language is context-free.

   (m) ________ Every context-free language is in $\mathcal{P}$.

   (n) ________ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

   (o) ________ The problem of whether a given string is generated by a given context-free grammar is decidable.

   (p) ________ If G is a context-free grammar, the question of whether $L(G) = \emptyset$ is decidable.

   (q) ________ Every context-free grammar is equivalent to some Chomsky normal form grammar.

   (r) ________ If L_1 is $\mathcal{NP}$ complete qand L_2 is $\mathcal{NP}$, there is a $\mathcal{P}$–TIME reduction from L_1 to L_2.

   (s) ________ If L_1 is $\mathcal{NP}$-hard and there is a $\mathcal{P}$–TIME reduction of L_1 to L_2, then L_2 must be $\mathcal{NP}$-hard.

   (t) ________ The partition problem is $\mathcal{NP}$ complete.

2. Give two context-free languages whose intersection is not context-free.

3. Identify which grammar generates each language. In each case, the language is over the binary alphabet $\Sigma = \{0, 1\}$.

   (a) The language of all palindromes of even length over Σ.

   (b) The language of all palindromes of odd length over Σ.

   (c) The language of all strings w over Σ such that $\#_0(w) = \#_1(w)$.

   (d) The language of all strings w over Σ such that $\#_0(w) = \#_1(w)$ and each prefix of w has at least as many zeros as ones.

   (e) The set of all binary numerals for multiples of three, where leading zeros are allowed.

   (i)
   $S \to 0S1S$
   $S \to \lambda$

   (ii)
   $S \to 0S0$
   $S \to 1S1$
   $S \to \lambda$

   (iii)
   $S \to 0S0$
   $S \to 1S1$
   $S \to 0$
   $S \to 1$

   (iv)
   $S \to 0S1S$
   $S \to 1S0S$
   $S \to \lambda$

   (v)
   $S \to 0S$
   $S \to 1A$
   $A \to 1S$
   $A \to 0B$
   $B \to 1B$
   $B \to 0A$
   $S \to 0$

4. The following grammar G_1 generates a simple algebraic language. Prove that G_1 is ambiguous by drawing two different parse trees for some string in $L(G_1)$. The start symbol of G_1 is E.

   1. $E \to E + E$
   2. $E \to E * E$
   3. $E \to (E)$
   4. $E \to x$
   5. $E \to y$
   6. $E \to z$

5. The grammar G_2, given below, is equivalent to G_1, but is unambiguous. Draw a parse tree for $(x + y) * z + y + x$ using G_2. (E stands for "expression," T stands for "term," and F stands for "factor.")

   1. $E \to E + T$
   2. $E \to T$
   3. $T \to T * F$
   4. $T \to F$
   5. $F \to (E)$
   6. $F \to x$
   7. $F \to y$
   8. $F \to z$

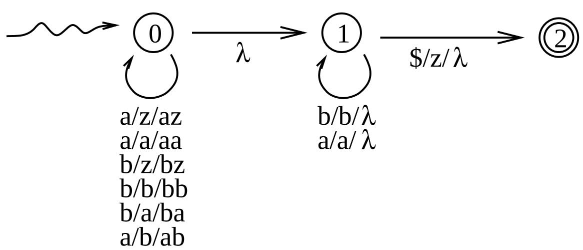
6. Consider the following list of languages. In each case, the alphabet is $\Sigma = \{a, b\}$.

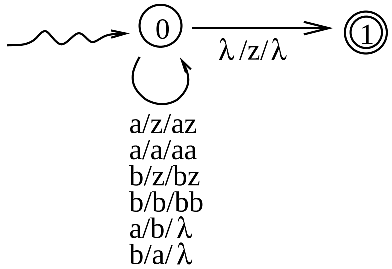
   (a) The language L_1 consisting of all strings which have an equal number of each symbol.

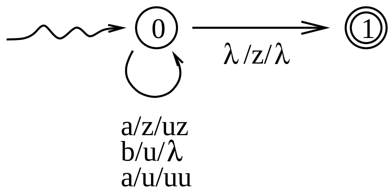
   (b) The language $L_2 \subseteq L_1$ where each prefix of any string in L_2 has at least as many $a's$ as $b's$.

   (c) The language of all even length palindromes.

   Each of the above languages is accepted by one of the PDA shown below. Which one?







7. A Chomsky normal form grammar (CNF grammar) is defined to be a context-free grammar with start symbol S such that:

   (a) The right side of any production is either two variables, one terminal, or the empty string.

   (b) S may not be on the right side of any production.

   (c) If the right side of a production is the empty string, the left side is S.

   Here is a simple example of a CNF grammar, which generates $\{a^n b^n \mid n \geq 0\}$:
   $S \to XB$
   $X \to AY$
   $Y \to XB$
   $A \to a$
   $X \to a$
   $B \to b$

   Let G be the following ambiguous CF grammar, that you have seen before:
   $S \to iS$
   $S \to iSeS$
   $S \to a$
   Construct a CNF grammar equivalent to G.