University of Nevada, Las Vegas Computer Science 456/656 Fall 2025

Answers to Assignment 1: Due Saturday January 25, 2025, 11:59 PM

Name:_____

You are permitted to work in groups, get help from others, read books, and use the internet. Turn in the assignment as instruced by the Teaching Assistant, Louis Dumontet louis.dumontet@unlv.edu.

1. Indicate which of the following sets are countable by **C**. If the set is uncountable, write **U**.

   (a) **C** $\mathbb{Z}$ the set of integers.

   (b) **C** $\mathbb{Q}$ the set of rational numbers.

   (c) **U** $\mathbb{R}$ the set of real numbers.

   (d) **C** $\{0.1\}^*$ The set of strings over the binary alphabet $\{0, 1\}$.

   (e) **U** $2^{\{0.1\}^*}$ The set of languages over the binary alphabet $\{0, 1\}$.

2. Prove that $\sqrt{2}$ is irrational.

   Theorem 1. *There is no rational number whose square is 2.*

   Proof. Assume that there is some fraction $\frac{p}{q}$ equal to the square root of 2.

   We assume that the fraction is reduced to the lowest temrs, *i.e.* p and q have no common divisor greater than 1. Then

$$\begin{aligned}
\sqrt{2} &= \frac{p}{q} \\
2 &= \frac{p^2}{q^2} \\
2q^2 &= p^2 \text{ thus } p^2 \text{ is even} \\
&\quad \text{Thus } p \text{ is even} \\
p &= 2k \text{ for some integer } k \\
p^2 &= 4k^2 \\
2q^2 &= 4k^2 \\
q^2 &= 2p^2 \text{ thus } q^2 \text{ is even} \\
&\quad \text{Thus } q \text{ is even}
\end{aligned}$$

   Since p and q are both even, they have a common divisor of 2, contradiction. Hence our assumption, that $\sqrt{2}$ is rational, is false. □