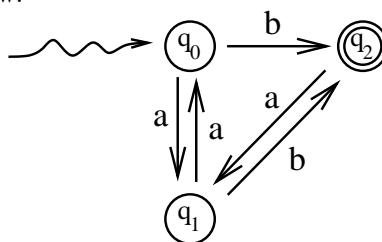
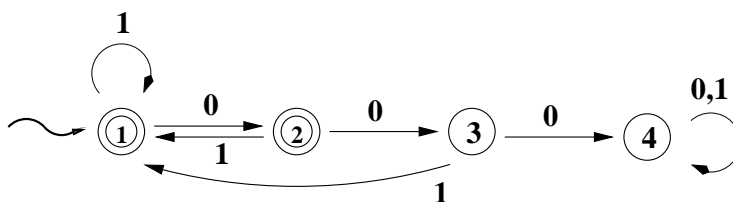


Answers to Assignment 2: Due Saturday February 1, 2025

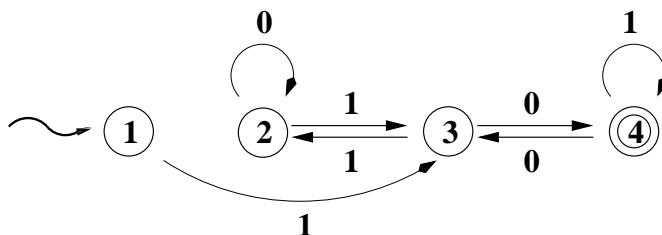
1. Let  $M_1$  be the DFA shown below.



Let  $M_2$  be the DFA shown below.



Let  $M_3$  be the DFA shown below.



Which of the following languages is accepted by  $M_1$ ? By  $M_2$ ? By  $M_3$ ?

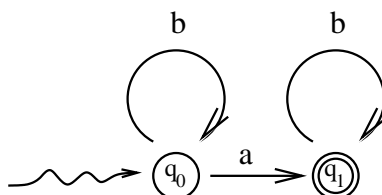
$M_1$  accepts (b).

$M_2$  accepts (a).

$M_3$  accepts (c).

- (a) The language of all binary strings in which every substring 00 is followed by 1.
- (b) All strings over  $\{a, b\}$  which end in b and which do not contain the substring bb.
- (c) The language of all binary numerals for positive integers equivalent to 2 modulo 3.
- (d) The language of all strings over  $\{a, b\}$  in which every b is followed by a.

Construct a DFA which accepts the language  $\{b^i a b^j : i, j \geq 0\}$ , the language of all strings over  $\{a, b\}$  which contain exactly one  $a$ . Your figure need not show the dead state.



2. Recall that  $\emptyset$  is the empty language. If  $L$  is some language, what is the concatenation  $\emptyset L$ ?

$\emptyset$

3. Let  $L_1 = \{\lambda\}$ . the language consisting of only the empty string. If  $L_2$  is some other language, what is the concatenation  $L_1 L_2$ ?

$L_2$

4. Is concatenation of languages commutative? That is, is the equation  $L_1 L_2 = L_2 L_1$  always true?

No. For example, if  $L_1 = \{a\}$  and  $L_2 = \{b\}$ , then  $L_1 L_2 = \{ab\}$  and  $L_2 L_1 = \{ba\}$ .

5. Which of the following is true:

- (a) If  $L$  is any language,  $L^0 = L$ .
- (b) If  $L$  is any language,  $L^0 = \emptyset$ .
- (c) If  $L$  is any language,  $L^0 = \{\lambda\}$ .

$L^0 = \{\lambda\}$

6. Does concatenation of languages distribute over union? That is, is  $L_1(L_2 + L_3) = L_1 L_2 + L_1 L_3$  always true?

Yes. To convince yourself, work out a few tiny examples.

7. What is  $\emptyset^*$ , the Kleene closure of the empty language?

$\{\lambda\}$ . See problem 5 above.

8. True(T) or False(F).

- i **F** Concatenation is commutative. That is,  $L_1 L_2 = L_2 L_1$  for any languages  $L_1$  and  $L_2$ .
- ii **T** Concatenation is associative. That is,  $(L_1 L_2) L_3 = L_1 (L_2 L_3)$  for any languages  $L_1$ ,  $L_2$ , and  $L_3$ .
- iii **T** The intersection of any two regular languages is regular.
- iv **T** The complement of any regular languages is regular.
- v **T** The Kleene closure of any regular languages is regular.

9. The DFA  $M_1$  shown in Problem 1 is not minimal, that is, it is equivalent to a DFA with fewer states. Can you draw a state diagram of that DFA? Your figure need not show the dead state.

