University of Nevada, Las Vegas Computer Science 456/656 Spring 2025

Answers to Assignment 4: Due Saturday March 1, 2025

Name:____

You are permitted to work in groups, get help from others, read books, and use the internet. You will receive a message from the graduage assistant, Louis, telling you how to turn in the assignment.

If w is any string, we write w^{-1} for the *inverse* of w. For example, if $w = abcca$ then $w^{-1} = accba$. A *palindrome* is a string which is equal to its own inverse, such as "level." We write $\#_0(w)$ for the number of zeros in w, and $\#_1(w)$ for the number of ones.

1. True or False. If the answwer is unknown to science at this time, write **O** for Open.
   (a) **F** Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.
   (b) **T** The union of any two context-free languages must be context-free
   (c) **T** The language $\{a^n b^n \mid n \geq 0\}$ is context-free, hence is accepted by a PDA.
   (d) **F** The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.
   (e) **T** The language $\left\{a^i b^j c^k \mid j = i + k\right\}$ is context-free.
   (f) **T** The intersection of any regular language with any context-free language is context-free.
   (g) **F** The intersection of any two context-free languages is context-free.
   (h) **T** If L is a context-free language over an alphabet with just one symbol, then L is regular.
   (i) **T** There is a deterministic parser for any context-free grammar.
   (j) **T** The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.
   (k) **T** The Kleene closure of any context-free language is context-free.
   (l) **T** Every regular language is context-free.
   (m) **T** Every context-free language is in $\mathcal{P}$.
   (n) **T** Every language accepted by a non-deterministic machine is accepted by some deterministic machine.
   (o) **T** The problem of whether a given string is generated by a given context-free grammar is decidable.
   (p) **T** If G is a context-free grammar, the question of whether $L(G) = \emptyset$ is decidable.
   (q) **T** Every context-free grammar is equivalent to some Chomsky normal form grammar.
   (r) **F** If L_1 is $\mathcal{NP}$ complete and L_2 is $\mathcal{NP}$, there is a $\mathcal{P}$–TIME reduction from L_1 to L_2.
   (s) **T** If L_1 is $\mathcal{NP}$-hard and there is a $\mathcal{P}$–TIME reduction of L_1 to L_2, then L_2 must be $\mathcal{NP}$-hard.
   (t) **T** The partition problem is $\mathcal{NP}$ complete.

2. Give two context-free languages whose intersection is not context-free.

   There are many examples. Here's one. Let $L_1 = \left\{a^i b^i c^j \;:\; i, j \geq 0\right\}$, and Let $L_2 = \left\{a^i b^j c^j \;:\; i, j, \geq 0\right\}$, Then $L1$, $L2$ are generated by the context-free grammars:

$S \to AC$	$S \to AC$
$A \to aAb$	$A \to aA$
$A \to \lambda$	$A \to \lambda$
$C \to Cc$	$C \to bCc$
$C \to \lambda$	$C \to \lambda$

   $L_1 \cap L_2 = \{a^n b^n c^n \;:\; n \geq 0\}$, which is not context-free.

3. Identify which grammar generates each language. In each case, the language is over the binary alphabet $\Sigma = \{0, 1\}$.

   (a) The language of all palindromes of even length over Σ.

   Generated by Grammar (ii).

   (b) The language of all palindromes of odd length over Σ.

   Generated by Grammar (iii).

   (c) The language of all strings w over Σ such that $\#_0(w) = \#_1(w)$.

   Generated by Grammar (iv).

   (d) The language of all strings w over Σ such that $\#_0(w) = \#_1(w)$ and each prefix of w has at least as many zeros as ones.

   Generated by Grammar (i).

   (e) The set of all binary numerals for multiples of three, where leading zeros are allowed.

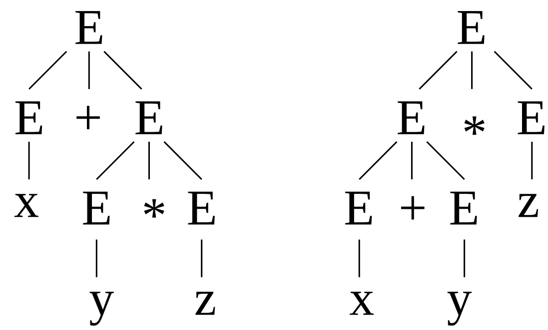
   Generated by Grammar (v).

(i)	(ii)	(iii)	(iv)	(v)
$S \to 0S1S$	$S \to 0S0$	$S \to 0S0$	$S \to 0S1S$	$S \to 0S$
$S \to \lambda$	$S \to 1S1$	$S \to 1S1$	$S \to 1S0S$	$S \to 1A$
	$S \to \lambda$	$S \to 0$	$S \to \lambda$	$A \to 1S$
		$S \to 1$		$A \to 0B$
				$B \to 1B$
				$B \to 0A$
				$S \to 0$

4. The following grammar G_1 generates a simple algebraic language. Prove that G_1 is ambiguous by drawing two different parse trees for some string in $L(G_1)$. The start symbol of G_1 is E.

   1. $E \to E + E$
   2. $E \to E * E$
   3. $E \to (E)$
   4. $E \to x$
   5. $E \to y$
   6. $E \to z$

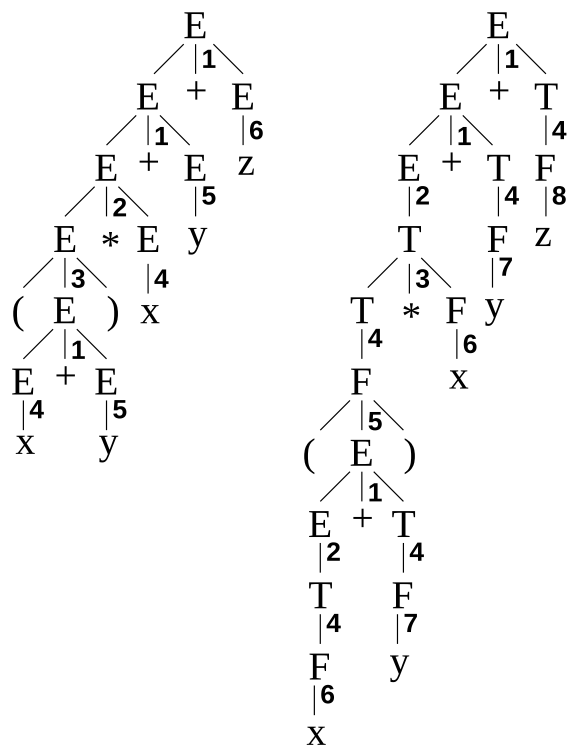
   There are infinitely many strings which have more than one parse tree. We choose $x + x * x$.



5. The grammar G_2, given below, is equivalent to G_1, but is unambiguous. Draw a parse tree for $w = (x + y) * z + y + x$ using G_2. (E stands for "expression," T stands for "term," and F stands for "factor.")

   1. $E \to E + T$
   2. $E \to T$
   3. $T \to T * F$
   4. $T \to F$
   5. $F \to (E)$
   6. $F \to x$
   7. $F \to y$
   8. $F \to z$

   Using G_1, the reverse rightmost derivation of w is 4513425161. Using G_2, the reverse rightmost derivation of w is 64274154632741841.



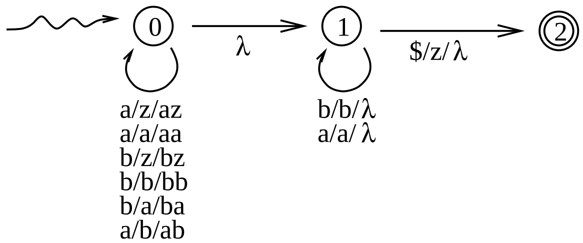
6. Consider the following list of languages. In each case, the alphabet is $\Sigma = \{a, b\}$.

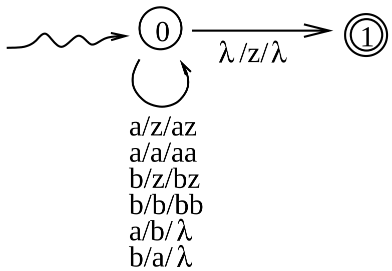
   (a) The language L_1 consisting of all strings which have an equal number of each symbol. Accepted by the second PDA below.

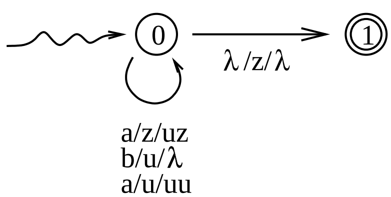
   (b) The language $L_2 \subseteq L_1$ where each prefix of any string in L_2 has at least as many $a's$ as $b's$. Accepted by the third PDA below.

   (c) The language of all even length palindromes. Accepted by the first PDA below.

   Each of the above languages is accepted by one of the PDA shown below. Which one?







7. A Chomsky normal form grammar (CNF grammar) is defined to be a context-free grammar with start symbol S such that:

   (a) The right side of any production is either two variables, one terminal, or the empty string.

   (b) S may not be on the right side of any production.

   (c) If the right side of a production is the empty string, the left side is S.

   Here is a simple example of a CNF grammar, which generates $\{a^n b^n \mid n \geq 0\}$:
   $S \to XB$
   $X \to AY$
   $Y \to XB$
   $A \to a$
   $X \to a$
   $B \to b$

Let G be the following ambiguous CF grammar, that you have seen before:
$S \to iS$
$S \to iSeS$
$S \to a$
Construct a CNF grammar equivalent to G.

Since S is not allowed on the right hand side of any production, we introduce the variable T which is equivalent to S, but which is allowed on the rhs.

$S \to IT$
$S \to a$
$S \to AB$
$T \to IT$
$T \to a$
$T \to AB$
$A \to IT$
$B \to ET$
$I \to i$
$E \to e$

Can you think of a better solution, that is, with fewer productions?